

# Convolutions and Group-Ring

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10:37 AM

$$\phi * \psi = \Phi(\Phi^{-1}(\phi) * \Phi^{-1}(\psi)) \quad \phi, \psi \in \text{Fun}(g)$$

$$\Phi^{-1}(\phi * \psi) = \Phi^{-1}(\phi) * \Phi^{-1}(\psi) \quad \text{in } \text{Fun}(G)$$

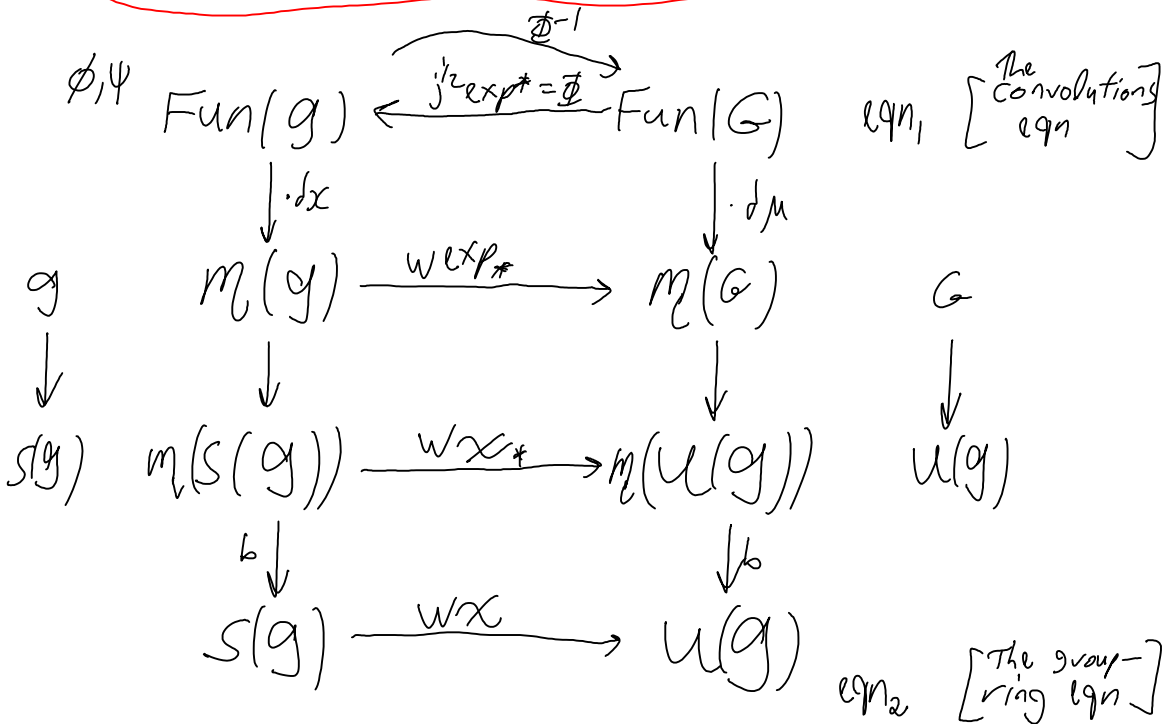
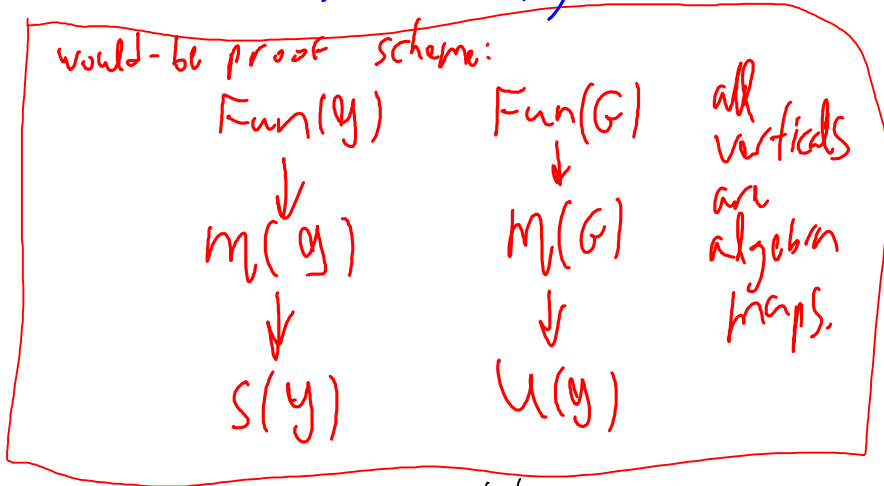


Diagram is commutative, verticals are algebra maps, horizontalals need not be so.

Convolutions and Group Rings (ignoring all Jacobians)			
CFL IF G is finite, $(\text{Fun}(G), *) \cong (\mathbb{R}G, \cdot)$ by $f \mapsto \sum f(a) \cdot a$	$(g, +)$ $\downarrow \exp$ $(G, \cdot)$	$\mathbb{R}g$ $\downarrow \chi$ $u(g)$	Embedding $x \mapsto e^x$ $\downarrow \exp$ $\downarrow \chi$ $e^x \mapsto e^x$

# Convolutions and Group Rings (ignoring all Jacobians)

IF  $G$  is finite,  
 CFL  $(\text{Fun}(G), *) \cong (\mathbb{R}G, \cdot)$  by  $f \mapsto \sum f(a) \cdot a$

$(G, +)$	$\mathbb{Z}G$	Embedding
$\downarrow \text{exp}$	$\downarrow \times$	$x \mapsto e^x$
$(G, \cdot)$	$\widehat{U}(G)$	$e^x \mapsto e^x$

$\psi_1, \psi_2 \in \text{Fun}(G) \rightsquigarrow$  Compare  $\mathcal{F}^{-1}(\psi_1) * \mathcal{F}^{-1}(\psi_2)$  with  $\mathcal{F}^{-1}(\psi_1 * \psi_2)$  in  $\widehat{U}(G)$ :

$(\mathcal{F}^{-1} \psi_i) = \int \psi_i(x) e^{x \cdot} dx \in \widehat{U}(G)$

convolution in  $G$ :

$\int \int \psi_1(x) \psi_2(y) e^{x \cdot} e^{y \cdot} dx dy$

convolution in  $\mathfrak{g}$ :

$\int \int \psi_1(x) \psi_2(y) e^{(x+y) \cdot} dx dy$

$\psi_i \in \text{Fun}(G)$	$\xrightarrow{\mathcal{F}^{-1}}$	$\widehat{S}(G)$
$\downarrow \mathcal{F}^{-1}$		$\downarrow \times$
$\text{Fun}(G)$	$\xrightarrow{\mathcal{F}}$	$\widehat{U}(G)$