Pulling back automorphisms

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2:18 PM

$$\begin{tikzcd} F_2 \arrow[r] & F_5 \arrow[loop above, "B"] \end{tikzcd}$$

pulling back automorphisms?

Nah; Perhaps the right thing to study are groups G, along with a pair of maps $\tau, \theta : F_n \to G$?

$$\begin{tikzcd}[column sep=small] F_n^1 \arrow[dr] & & F_n^2 \arrow[dl] \arrow[dr] & & F_n^3 \arrow[dl] \\ & G_1 \arrow[dr] & & G_2 \arrow[dl] & \\ & & G_1 *_{F_n^2} G_2 & & \end{tikzcd}$$

Maybe it is $\{\text{Quotients of } F_n\}$, I.e., $F_n \twoheadrightarrow G$
with the operations

$$(G_1, G_2) \longmapsto G_1 \times G_2$$

and pullbacks — compositions — with maps

$$F_m \longrightarrow F_n$$

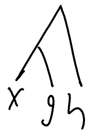
and also, imposing the relations $x_i = x_j$, for any pair of generators x_i & x_j (and reduction to the image of F_{n-2}?) ?

Nyet; I should be talking about "groups with a peripheral system".

Question - what is the finite type completion of the free quandle on some generators?

Quandle:

$$(x\uparrow g)\uparrow h = (x\uparrow h)\uparrow(g\uparrow h)$$

 $= \dots\,.$

In groupland this is

$$h^{-1}g^{-1}xgh = (h^{-1}g^{-1}h)(h^{-1}xh)(h^{-1}gh)$$

$$x\uparrow g - 1 = x\uparrow g - x + x - 1$$
$$= x^{g-1} + (x-1) \underset{\text{mod deg } 2}{\equiv} 1^{g-1} + (x-1) = x-1$$

so $\overline{x\uparrow g} = \bar{x}$

In a group

$$(g_1 g_2 - 1) = g_1(g_2 - 1) + (g_1 - 1)$$

so $\overline{g_1 g_2} = \bar{g}_2 + \bar{g}_1$

Guess: $[\bar{x},\bar{y}] := \bar{x}\uparrow\bar{y}$ makes a Lie algebra, or at least a Leibnitz algebra, whatever that may be.

$$\big((\bar{x}+1)\uparrow(\bar{g}+1)\big)\uparrow(\bar{h}+1) = \big((\bar{x}+1)\uparrow(\bar{h}+1)\big)\uparrow\big((\bar{g}+1)\uparrow(\bar{h}+1)\big)$$

deg 0: $1=1$ deg 1: $\bar{x}=x$

deg 2: $\bar{x}\uparrow\bar{g} + \bar{x}\uparrow\bar{h} = \bar{x}\uparrow\bar{h} + \bar{x}\uparrow\bar{g}$

deg 3: $(\bar{x}\uparrow\bar{g})\uparrow\bar{h} = (\bar{x}\uparrow\bar{h})\uparrow\bar{g} + \bar{x}\uparrow(\bar{g}\uparrow\bar{h})$

The Jacobi relation!

There's also $(1+\bar{x})\uparrow(1+\bar{x}) = 1+\bar{x}$

deg 0: $1=1$ deg 1: $\bar{x}=\bar{x}$

deg 2: $\bar{x}\uparrow\bar{x} = 0$

(bilinear & vanishes on diagonal means anti-symmetric: $0 = (x+y)\uparrow(x+y) = x\uparrow x + y\uparrow y + x\uparrow y + y\uparrow x$
$= x\uparrow y + y\uparrow x$)

An expansion: try $x \longmapsto 1+\bar{x}$ for the generators,
$1 \longmapsto 1$

An expansion: $\overset{\text{try}}{x} \longmapsto 1+\bar{x}$ for the generators,
$1 \longmapsto 1$

$$\begin{tikzcd}
(x,y) \arrow[r, "\uparrow"] \arrow[d, "Z"'] & x\uparrow y \arrow[d, "Z"] \\
(1+\bar{x}, 1+\bar{y}) \arrow[r, "\uparrow"] & (1+\bar{x})\uparrow(1+\bar{y}) \\
& (1+\bar{x})\uparrow(1+\bar{y}) \arrow[u, phantom, "//"]
\end{tikzcd}$$

Does it annihilate the quandle relation?

$$(x\uparrow g)\uparrow h \quad = \quad (x\uparrow h)\uparrow(g\uparrow h)$$
$$\downarrow$$

The only remaining term is in deg 4: $\quad 0 = (\bar{x}\uparrow\bar{h})\uparrow(\bar{g}\uparrow\bar{h})$

$$[[\bar{x},\bar{h}],[\bar{g},\bar{h}]] \qquad \text{fails!}$$

<u>Question</u> Let G be a group and Q its group-quandle. Is it true that
$$\mathcal{U}(Q^{ft}) = G^{ft} \quad ?$$