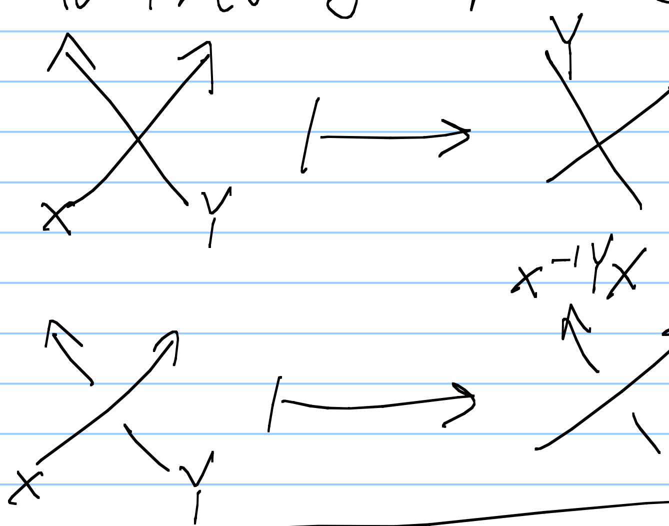
The infinitesimal Artin Rep. for virtuals

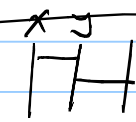
Note Title 14/02/2008

<u>Question:</u> Virtual braids induce automorphisms of the free group via

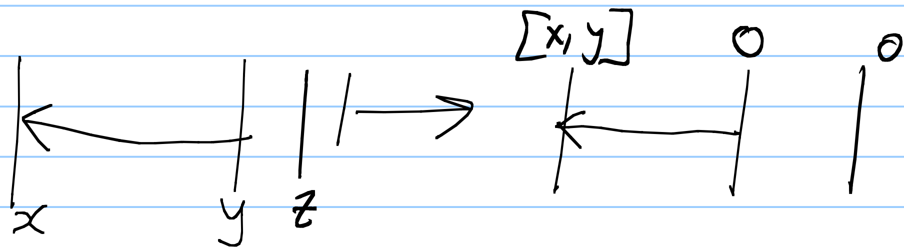


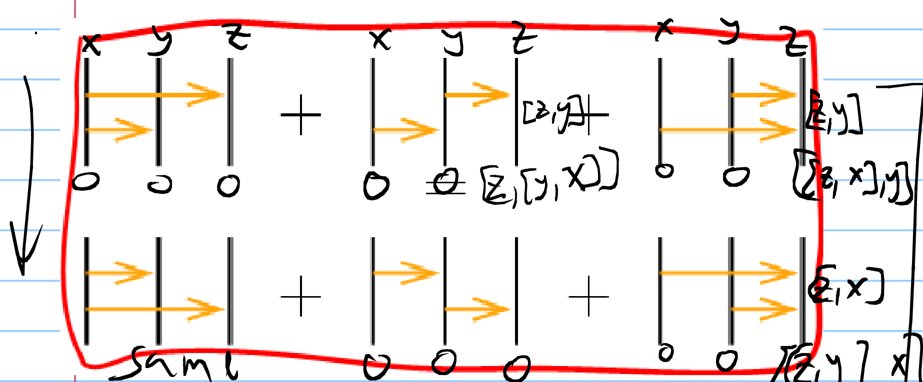
what is the infinitesimal version of this map

$VB_n \to Aut(F_n)$?

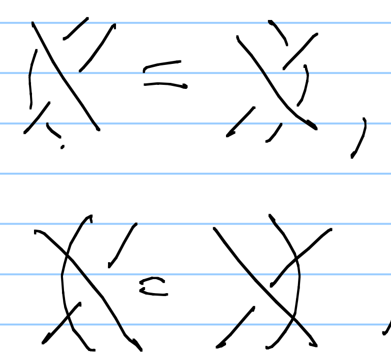


Sol'n: Probably,

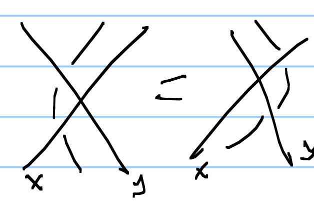




Virtual Reidemeister:



but not



Note Title 13/02/2008

$\phi : g \longmapsto g^{-1} X g$ $\quad$ $(g^{-1}-1)(X-1) + g^{-1}(X-1)(g-1)$

$$\begin{aligned} &\qquad\qquad\qquad\qquad\quad \| \\ g^{-1}Xg - X &= (g^{-1}-1)X + g^{-1}X(g-1) = -\bar{g} + \bar{g} = 0 \end{aligned}$$

So $\bar{\phi}\big|_{\deg 1} = 0.$

$\overline{\Psi_p}$ is not multiplication by p!

$$\begin{array}{ccccc} X_1 X_2 & \xmapsto{Z^M} & (1+x_1)(1+x_2) & \xrightarrow{\overline{\Psi_p}} & 1 + p x_1 + p x_2 + \frac{(p+1)p}{2} x_1 x_2 + \frac{(p-1)p}{2} x_2 x_1 \\ \Big\downarrow \overline{\Psi_p} & & & & \\ (X_1 X_2)^p & \xmapsto{Z^M} & ((1+x_1)(1+x_2))^p = & & \end{array}$$

agrees up to degree 2, but then diverges.

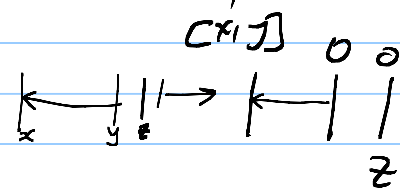
$$X_1 X_2 \xmapsto{Z^{Exp}} e^{x_1} e^{x_2} \xrightarrow{\Psi_p} (e^{x_1} e^{x_2})^p$$

So Ψ_p is compatible with Z^{Exp}.

<u>Question</u> Consider the derivation

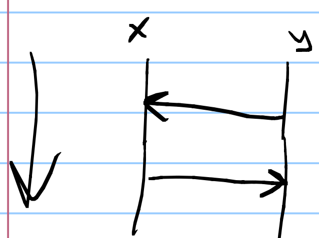
$$x \longmapsto [x,[y,[x,y]]] \qquad \text{on}$$
$$y \longmapsto -[y,[x,[x,y]]] \qquad FL(x,y)$$

is it in the range of the infinitesimal virtual Artin?



$x+y \mapsto 0$ as

$$0 = [[x,y],[x,y]] = [x,[y,[x,y]]] - [y,[x,[x,y]]]$$



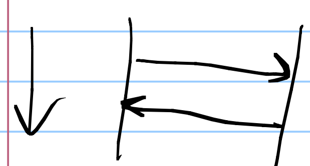
: $x \longmapsto [x,y] \longmapsto [x,[y,x]] = -[x,[x,y]]$

$y \longmapsto 0$

$[x,y] \mapsto [[x,y],y]$

$\mapsto [[x,[y,x]],y]$

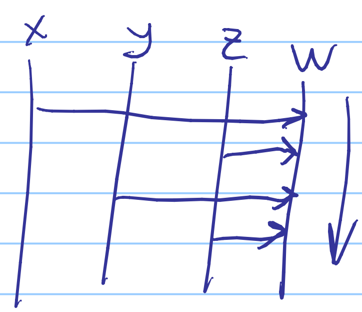
$\cancel{+[[x,y],[y,x]]}$



: $x \longmapsto 0$

$y \longmapsto [y,x] \longmapsto [y,[x,y]]$

What is



?

<u>Ans:</u>

$x,y,z \longmapsto 0$

$w \mapsto [w,x] \longmapsto$

$[[w,z],x] \longmapsto [[[w,y],z],x]$

etc.

So it seems that every ("divergence free") tangential derivation is in the range of the infinitesimal virtual Artin; Though there must be a kernel....

So it seems like the following is true:

__Claim__ The Lie algebra $TDer_n$ of tangential derivations of the free Lie algebra (That was wishful thinking!) oops, what about $[\mapsto, \mapsfrom]$?

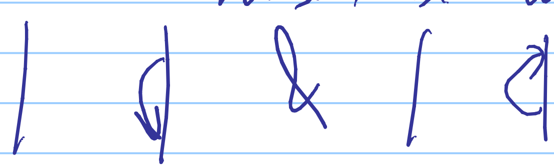
__Question__ Is the derivation

$$x \longmapsto [x,[x,y]]$$
$$y \longmapsto 0$$

(on $FL(x,y)$)

in the image of IVA_2 ? (IVA = Infinitesimal virtual Artin)

Probably not.

__Question__ Is there a reasonable way to incorporate  ?

Question Pure tangles induce automorphisms of the nilpotent completion of the free group. What is the chord-diagram analogue of that?

Answer: The last strand, representing an element of the free group, should be taken modulo homotopy:

The usual question: What's the virtual analog?