Is there a Hilbert's Nullstellensatz for finite type invariants of links?

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Let *k* be an algebraically closed field and let *I* be an ideal in the polynomial ring $k[x_1,...,x_n]$. The Hilbert Nullstellensatz (see e.g. [E]) says that the ideal of polynomials in $k[x_1,...,x_n]$ that vanish on the variety defined by the common zeros of all polynomials in *I* is the radical of *I*.

Problem: Is there a similar statement for finite type invariants of links? Let *I* be an ideal in the algebra *V* of finite type invariants of links. Let *Z* be the set of links that are annihilated by all members of *I*, and let *J* be the ideal in *V* of all invariants that vanish on *Z*. Clearly, *J* always contains the radical of *I*. Are they always equal?

Example: Let *I* be the ideal generated by linking numbers. In this case, *Z* is the set of algebraically split links. Is it true that every finite type invariant that vanishes on algebraically split links is a sum of multiples of linking numbers by arbitrary other finite type invariants of links? I believe it is true, and I believe it follows from the results of Appleboim [A], but I'm afraid Appleboim's paper is incomplete and while I believe it I cannot vouch for its validity.

Remark: One may also ask, "what is the Zariski closure of a given set of links?". I believe that in the light of the paragraphs above the meaning of this question should be clear. I know of at least one interesting example: In [N] Ng shows that the Zariski closure of the set of ribbon knots is the set of knots whose Arf invariant vanishes.

References:

[A]
E. Appleboim, *Finite type invariants of links with fixed linking matrix,* arXiv:math.GT/9906138.

[E]
D. Eisenbud, *Commutative Algebra With a View Toward Algebraic Geometry,* Graduate Texts in Mathematics **150**, Springer-Verlag, 1994.

[N]
K. Y. Ng, *Groups of ribbon knots,* arXiv:q-alg/9502017.