Date 10.27.14 No.

Read Along 2.1-2.3 term test ~~returned~~ discussed

@ 9:50

Riddle Along "The game of 15" $\boxed{1}\ \boxed{2}\ \boxed{3}\ \boxed{4}\ \boxed{5}\ \boxed{6}\ \boxed{7}\ \boxed{8}\ \boxed{9}$

2 players alternate drawing cards from the above deck.
The 1st to have within her cards 3 that add up to 15. wins.
Would you like to move 1st/2nd? Link to video on today's web

Today Linear Transportation abstractly

Reminder $V, W/F$; $\mathcal{L}: V \to W$ "Linear Transformation"
if it preserves structures $\Leftrightarrow \mathcal{L}(\sum \alpha_j u_j) = \sum \alpha_j \mathcal{L}(u_j)$

def: $\mathcal{L}(V, W) := \{$the set of all linear transformation $V \to W\}$

catigraphic form $\subset \mathcal{F}(V, W)$

Proposition $\qquad$ a vector space

$\mathcal{L}(V, W)$ is a vector space ~~with F~~ over the field F

0.

$\mathcal{L}(V,W) \ni 0_{\mathcal{L}(V,W)}$ defined by $\forall u \in V$
$0_{\mathcal{L}(V,W)}(u) = 0_W$

1. $T, S \in \mathcal{L}(V, W)$ $\quad T+S: V \to W$ ~~Addition~~ inside L.T.
$u \in V \quad (T+S)(u) = T(u) + S(u)$

2. $(cT)(u) = c \cdot T(u)$ $\qquad$ L is a L.t. if $L(cu+v) = cL(u) + L(v)$

Pf $\square$

If T, S are l.t., is T+S really a l.t.?

Check: $(T+S)(cu+v) \overset{?}{=} c(T+S)(u) + (T+S)(v)$

$LHS = T(cu+v) + S(cv+v) = cT(u) + T(v) + cS(u) + S(v)$

No.

Date

$RHS = c(T(u) + S(u)) + T(v) + S(v) =$

$\hookrightarrow = cT(u) + cS(u) + T(v) + S(v)$

$LHS = T(cu+v) + S(cu+v) = cT(u) + T(v) + cS(u) + cS(v)$

$LHS = RHS$! (check l.t.)

<u>Claim</u> The "composition" of l.t. is an l.t.

$X \xrightarrow{f} Y \xrightarrow{g} Z \quad x \in X,$

"the comp. of f & g"

$(g \circ f)(x) = g(f(x))$

<u>Claim</u> e.g. If $U \xrightarrow{T} V \xrightarrow{S} W$

|| $U, V, W / F$, $T: U \to V$ & $S: V \to W$ are l.t.

Then $S \circ T: U \to W$ is a l.t.

|| ST "multiplication for l.t. not always defined"

<u>PF</u> $(ST)(cx+y) = S(T(cx+y)) =$

$S(c\underbrace{T(x)}_{u} + \underbrace{T(y)}_{v}) = cS(T(x)) + S(T(y))$

$= c(ST)(x) + (ST)(y)$ $\qquad TS \neq ST$

e.g. $U = V = W = P(\mathbb{R})$

$T = D: P(\mathbb{R}) \to P(\mathbb{R}) \quad Df = \frac{d}{dx} f = f'$ (momentum)

$S = \hat{x}: P(\mathbb{R}) \to P(\mathbb{R}) \quad \hat{x} f = x \cdot f$ (position)

$TS \neq TS$ not communitive include $(TS)(x^k)$

$(ST)(x^k) = S(kx^{k-1}) = \underbrace{kx^k}_{} \neq TS(x^k) = T(x^{k+1}) = \underbrace{(k+1)x^k}_{}$

(key in quantum mechanics)

<u>Loosely</u> A l.t. is fully determined by its values on a basis. Furthermore, these values can be arbitrary.

22Lines. 8mm

Date No.

<u>Precisely</u> <u>Thm</u> If $V, W/F$, if $\beta = (u_1, \dots, u_n)$ is a basis of V and if $w_1, \dots w_n$ are any elements of W, then $\exists!\, L: V \to W$ satisfies $\forall i$ $L(u_i) = w_i \Rightarrow \mathcal{L}(V, W) \cong \{(w_1, \dots w_n)\}$

<u>Pf</u> Given w_i, set $L(u) = \left(\sum \alpha_j L(u_j)\right) = \sum \alpha_j w_i$

whenever $u = \sum \alpha_j u_j$

Now check that L is indeed a l.t. satisfies $L(u_i) = w_i$ $\Box$