MAT240 · Tutorial — Nov 16, 2004.

$\boxed{3.2}$ 19. $\text{rank}(AB) = \text{rank}(L_{AB}) = \text{rank}(L_A L_B)$

$$\mathbb{F}^p \xrightarrow{L_B} \mathbb{F}^n \xrightarrow{L_A} \mathbb{F}^m$$

L_B onto

$\Rightarrow \text{Im}\, L_B = \mathbb{F}^n$

$\Rightarrow \text{Im}\, L_A L_B = \text{Im}\, L_A = \mathbb{F}^m \because L_A$ is onto.

$\boxed{2.4}$ 9. L_{AB} invertible $\quad \mathbb{F}^n \xrightarrow{L_{AB}} \mathbb{F}^n$

$\overset{\shortparallel}{L_A L_B}$

① L_{AB} is onto $\Rightarrow L_A$

$\Rightarrow L$ is an iso

$\Rightarrow A$ B invertible

② L_{AB} is 1-1 $\Rightarrow L_B$ is 1-1 $\xrightarrow{B}$.

p.180 2.c) nullspace $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 1 \end{bmatrix} = \text{nullspace}\begin{pmatrix} 1 & 2 & -1 \\ 0 & -3 & 3 \end{pmatrix} = \text{nullspace}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$

b/c solving for $=0$.

$= \left\{ (x_1, x_2, x_3) \;\middle|\; \begin{matrix} x_1 + x_3 = 0 \\ x_2 - x_3 = 0 \end{matrix} \right\} = \{(-x_3, x_3, x_3)\} = \text{span}\{(-1,1,1)\}$

↳ basis. dim = 1.

3. c) $(3,0,0)$ is a particular sol'n. All sol'ns are given by $(3,0,0) + t(-1,1,1) \quad t \in \mathbb{R}$.

6. Can define $T^{-1}(1,11) = \{(a,b,c) \mid T(a,b,c) = (1,11)\}$

$a + b = 1$

$2a - c = 11$

Find general sol'n to $\begin{cases} a+b=0 \\ 2a-c=0 \end{cases}$

Then find particular sol'n to $\begin{cases} a+b=1 \\ 2a-c=11 \end{cases}$

4)
P.107

5. A is invertible $\Leftrightarrow \exists B$ s.t. $AB = BA = I$.
$\Leftrightarrow B^t A^t = A^t B^t = I^t = I$
$\Leftrightarrow (A^t)^{-1} = B^t = (A^{-1})^t$

R(4)
2.3

~~W~~
16 a) $V \xrightarrow{T} V \xrightarrow{T} V$
dim formula: $\dim(N(T^2)) + \dim(R(T^2)) = \dim V$
$n(T^2) + r(T^2) = \dim V$
$n(T) + r(T) = \dim V$

$\Rightarrow n(T^2) = n(T)$ (*)
$\because T(v) = 0 \Rightarrow T^2(v) = 0$.
$N(T) \subseteq N(T^2)$
by (*) $N(T) = N(T^2)$
Let $v \in R(T) \cap N(T)$
~~⇒ ...~~
$\Rightarrow v = T(u)$ for some u
$T(v) = 0$.
$\Rightarrow T^2(u) = 0 \Rightarrow u \in N(T^2) = N(T)$
$\Rightarrow v = T(u) = 0$.
$\therefore R(T) \cap N(T) = \{0\}$.
$\square$

$R(T) \oplus N(T) \subseteq V$
They are equal by dim ~~...~~ formula.

b) rank $T \geq$ rank $T^2 \geq$ rank $T^4 \geq \cdots \geq$ rank$(T^{2^k}) \geq$.
$\because V$ is finite-dimensional the eventually, $\exists N$
~~...~~ rank$^{2^k}$ is constant for ~~...~~ $k \geq N$ large enough
~~...~~
So rank T^{2^N} = rank $T^{2^{N+1}}$ = rank$(T^{2^N})^2$
By part a) $V = R(T^{2^N}) \oplus N(T^{2^N})$

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$$A \quad \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

Write as product of elementary matrices

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R3-R1} \quad \xrightarrow{-1R3} \quad \xrightarrow{R_2-R_1} \quad \rightarrow$$

multiply A by

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{bmatrix} \cdot \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix} = I$$

then move E's to right & get answer.