Non Commutative Gaussian Elimination @ MAT 1100

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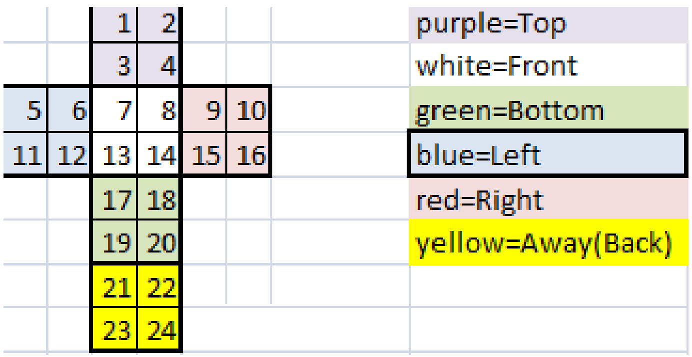
Amended from a similar notebook by Dror Bar-Natan and Itai Bar-Natan. The original version is at http://www.math.toronto.edu/~drorbn/Misc-/SchreierSimsRubik/.

HW1, Part I
Solving the 2x2x2 Rubik's Cube
Starting point:



Program 0

In[143]:=
```
(* generators are computed as clockwise 90 degrees rotations  when facing the respective face *)
gs = {
     purple = P[3, 1, 4, 2, 7, 8, 9, 10, 24, 23, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 6, 5]
   , white  = P[1, 2, 12, 6, 5, 17, 13, 7, 3, 10, 11, 18, 14, 8, 4, 16, 15, 9, 19, 20, 21, 22, 23, 24]
   , green  = P[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 22, 21, 11, 12, 13, 14, 19, 17, 20, 18, 16, 15, 23, 24]
   , blue   = P[21, 2, 23, 4, 11, 5, 1, 8, 9, 10, 12, 6, 3, 14, 15, 16, 7, 18, 13, 20, 17, 22, 19, 24]
   , red    = P[1, 8, 3, 14, 5, 6, 7, 18, 15, 9, 11, 12, 13, 20, 16, 10, 17, 22, 19, 24, 21, 2, 23, 4]
   , yellow = P[10, 16, 3, 4, 2, 6, 7, 8, 9, 20, 1, 12, 13, 14, 15, 19, 17, 18, 5, 11, 23, 21, 24, 22]
  };
(*gs={
       purple=P[3,1,4,2,7,8,9,10,24,23,11,12,13,14,15,16,17,18,19,20,21,22,6,5]
   ,red       =P[1,8,3,14,5,6,7,18,15,9,11,12,13,20,16,10,17,22,19,24,21,2,23,4]
   ,yellow=P[10,16,3,4,2,6,7,8,9,20,1,12,13,14,15,19,17,18,5,11,23,21,24,22]
  };*)
```

In[144]:=
```
gs
```

Out[144]=
```
{P[3, 1, 4, 2, 7, 8, 9, 10, 24, 23, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 6, 5],
 P[1, 2, 12, 6, 5, 17, 13, 7, 3, 10, 11, 18, 14, 8, 4, 16, 15, 9, 19, 20, 21, 22, 23, 24],
 P[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 22, 21, 11, 12, 13, 14, 19, 17, 20, 18, 16, 15, 23, 24],
 P[21, 2, 23, 4, 11, 5, 1, 8, 9, 10, 12, 6, 3, 14, 15, 16, 7, 18, 13, 20, 17, 22, 19, 24],
 P[1, 8, 3, 14, 5, 6, 7, 18, 15, 9, 11, 12, 13, 20, 16, 10, 17, 22, 19, 24, 21, 2, 23, 4],
 P[10, 16, 3, 4, 2, 6, 7, 8, 9, 20, 1, 12, 13, 14, 15, 19, 17, 18, 5, 11, 23, 21, 24, 22]}
```

In[145]:=
```
($RecursionLimit = 2^16;
  n = 24;
  P /: p_P ** P[a___] := p[[{a}]];
  Inv[p_P] := P @@ Ordering[p];
  Feed[P @@ Range[n]] := Null;
  (*Feed*)
  Feed[p_P] := Module[{i, j},
    For[i = 1, p[[i]] == i, ++i]; j = p[[i]];
    If[Head[s[i, j]] === P,
     Feed[Inv[s[i, j]] ** p],
     (*Else*) s[i, j] = p;
     Do[If[Head[s[k, l]] == P,
       Feed[s[i, j] ** s[k, l]];
       Feed[s[k, l] ** s[i, j]]
      ],
      {k, n}, {l, n}]
    ]]
 );

(Feed[#]; Product[1 + Length[Select[Range[n], Head[s[i, #]] === P &]], {i, n}]) & /@ gs
```

Out[125]= {27 978 373 094 031 360 000, 27 978 373 094 031 360 000, 27 978 373 094 031 360 000, 27 978 373 094 031 360 000, 27 978 373 094 031 360 000, 27 978 373 094 031 360 000}

In[137]:=
```
Images[i_] := {i}~Join~Select[Range[n], Head[s[i, #]] === P &];

ListPlot[
 Join @@ Table[{i, #} & /@ Images[i], {i, n}],
 AspectRatio → 1
]
```

Out[138]=