

$$\Rightarrow RHC = \text{span}((G/K) \cup L) = V \quad \square$$

Corollary 1. If V has a finite basis, then every other basis is also finite and has the same # of elements.

$\Rightarrow \dim V$ makes sense

Corollary 2. Assume $\dim V = n$ then

(a) If G generates V , then $|G| \geq n$ & if $|G| = n$ then it is a basis.

(b) If L is a linearly independent subset of V then $|L| \leq n$. If further $|L| = n$, then L is a basis, and furthermore if $|L| < n$, then L can be extended to a basis of V .

Proof of (a). Let $B = \{u_1, \dots, u_n\}$ of V by replacement can find $R \in G$ s.t. $|R| = n$ s.t. $(G/K) \cup B = V$

$\Rightarrow |G| \geq n$. & if $|G| = n$ then it is linearly independent. Indeed if it wasn't then there is some non-trivial linear combination equal to 0, in G , so an element of G is a linear combination of the others so G has a ~~sub~~ strict subset but still generates, but that can't be because every generating set has at least n elements.