2. If $A \in M_{n \times n}$ is invertible, and A' is its rref. Then

$\text{rank}(A) = \dim R(A) = \dim F^n = n$

$\Rightarrow$ # of pivots in A' is n

$\Rightarrow A' = \underbrace{\begin{pmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{pmatrix}}_{n} \Rightarrow A' = I_n$

$\Rightarrow$ our matrix invertible algorithm works!

A system of lin. eqn's:

$2x - 7y = -3$

$2y - x = 0.$

$x = ?\ y = ?$

In general, m eqn's w/ n unknowns

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$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_n.$$

$\Updownarrow$

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \quad x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \quad b = \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}$$

$$Ax = b$$

$$A = \begin{pmatrix} 2 & -7 \\ 2 & -1 \end{pmatrix} \quad \vec{x} = \begin{pmatrix} x \\ y \end{pmatrix} \quad b = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$$

$$A\vec{x} = b?$$

lucky case:

A is invertible.

$$Ax = b$$
$$\Updownarrow$$
$$x = A^{-1}Ax = A^{-1}b \iff x = A^{-1}b$$

$$A^{-1} = \begin{pmatrix} -\frac{2}{3} & -\frac{7}{3} \\ -\frac{1}{3} & -\frac{2}{3} \end{pmatrix}$$

$$\vec{x} = \frac{1}{3}\begin{pmatrix} -2 & -7 \\ -1 & -2 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$x=2$ $y=1$.

$Ax=b$ $A \in M_{m\times n}$ $b \in F^m$, $x \in F^n$

$b=0$. $Ax=0$ "homogeneous"

$b\neq 0$ $Ax=\begin{pmatrix} \neq \\ 0 \end{pmatrix}$ "inhomogeneous"

Homo case:

1. x is a solution $\iff$ $x \in N(T) / \mathrm{Ker}(T)$
2. sol'ns form a subspace of F^n