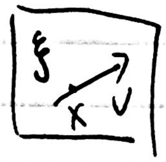
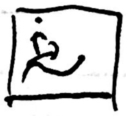
Feb 17th Fri. Hour 055

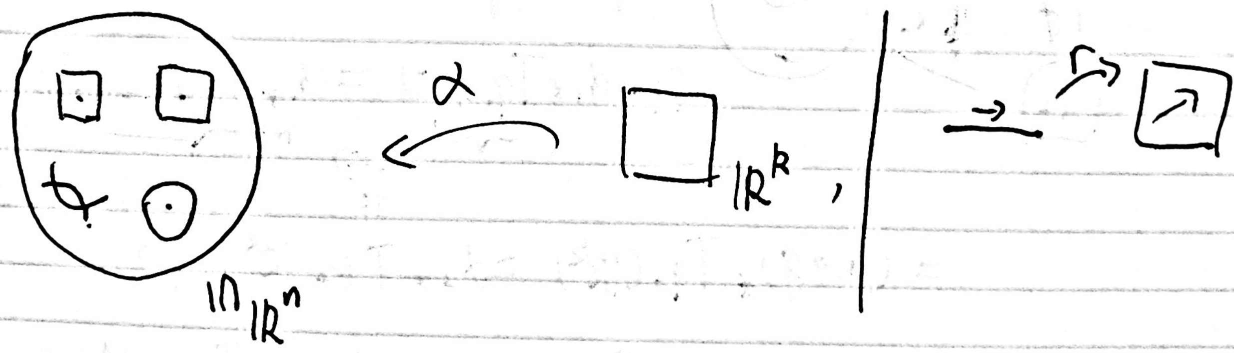
Tangent vector : $\xi = (x, 0)$

$T_x(\mathbb{R}^n) = \{\xi = (x, v)\}$ a vector space with open sets acting only on the v part

$t \longrightarrow \xrightarrow{r}$  $\dot{r}(t) = (r(t), D_{r(t)}(e_1))$

 $\xrightarrow{F} \mathbb{R}$ $\quad D_\xi F := D_{F_x}(v)$

$t \longrightarrow \xrightarrow{r}$  $\xrightarrow{F} \mathbb{R} \Rightarrow \dfrac{d(F \circ r)}{d(t)} = D_{\dot{r}(t)} F$



$\alpha : \mathbb{R}^k \to \mathbb{R}^n$, stand assumption α is of C^r.

$(x, v) = \xi \in T_x(\mathbb{R}^k)$, $(\alpha_* \xi) \in T_{\alpha(x)} \mathbb{R}^n$

$\qquad = (\alpha(x), D_{\alpha(x)} \cdot v)$

$$\begin{tikzcd} \mathbb{R}^k \arrow[r, "\alpha"] \arrow[rr, bend left, "\beta \circ \alpha"] & \mathbb{R}^n \arrow[r, "\beta"] & \mathbb{R}^m \end{tikzcd}$$

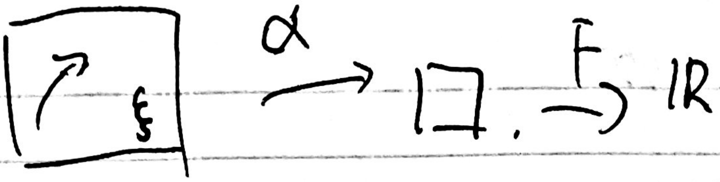
claim: $\beta_* \alpha_* \xi = (\beta \circ \alpha)_* \xi$

$\xi \to \alpha_* \xi \to \beta_*(\alpha_* \xi)$

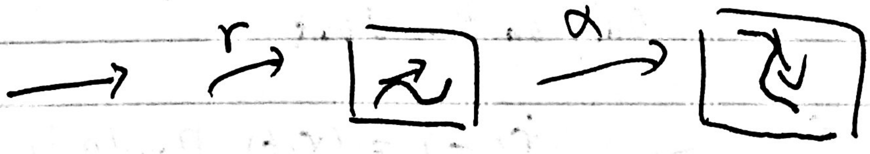
$\beta_* \alpha_* = (\beta \circ \alpha)_*$

$(\beta \circ \alpha)_* \xi$

Pf: chain rule.

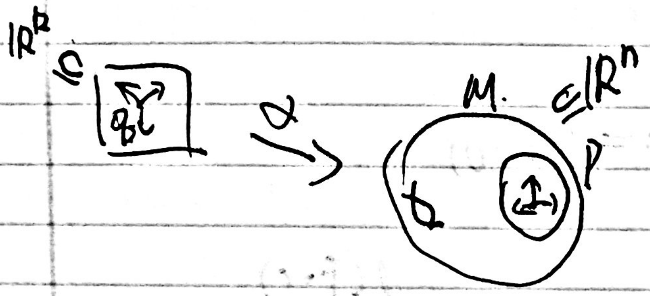


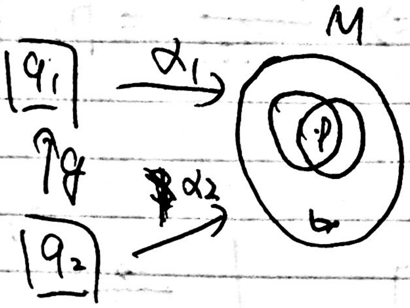
Claim: $D_\xi(\alpha^* F) = D_{\alpha_* \xi} F$ (chain rule)



Claim: $\alpha_*(\dot{r}(t)) = (\alpha \circ r)^{\cdot}(t)$

Def: $T_pM = \alpha_*(T_q\mathbb{R}^k) \subset T_p\mathbb{R}^n$





Claim: $\alpha_{1*} T_{q_1}\mathbb{R}^k = \alpha_{2*} T_{q_2}\mathbb{R}^k$

~~g is~~ g is a diffeo, $(Dg)(q_2)^\#$ is invertible

Pf: $\underbrace{\alpha_{1*} T_{q_1}(\mathbb{R}^k)} = \underbrace{\alpha_{1*} g_* T_{q_2}(\mathbb{R}^k)}$

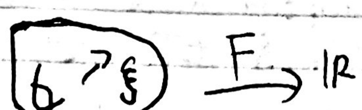
$\Longleftrightarrow$

$\#$

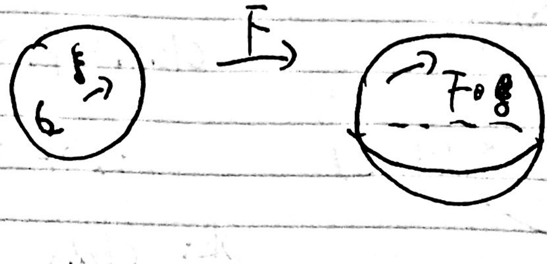
$= (\alpha_1 \circ g)_* T_{q_2}(\mathbb{R}^k) = \alpha_{2*} T_{q_2}\mathbb{R}^k$ □.

Comments: 1. 

$\dot{r}(t) \in T_{r(t)}M$. patch is ~~manifold~~ Manifold

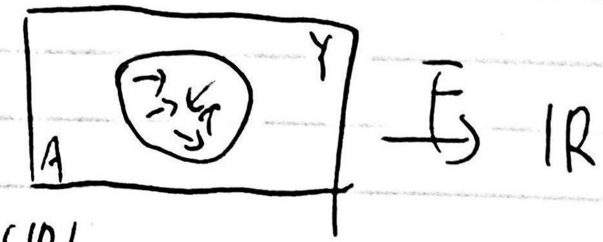
2. 

$D_\xi F$

3. 

C^r vector fields: $A \subseteq \mathbb{R}^n$ open.

$Y: A \to \bigcup_x T_x(\mathbb{R}^n),\ x \in A$



Such that $\forall x \in A$, $Y(x) \in T_x(\mathbb{R})$

In practice, $Y(x) = (x, \sum Y_i x e_i)$. $Y \longleftrightarrow$ function $\mathbb{R}^n \to \mathbb{R}^n$

{functions on A} $\xleftrightarrow{Y}$ {functions on A}

$\psi_F \longrightarrow (YF)(x) = D_{Y(x)}F$

Def: A vector field Y is called C^r if:

1. $\forall i$, Y_i is of C^r
2. $\forall F \in C^{r+1}$, $YF \in C^r$.