November 5th, 2009

MAT240: Abstract Linear Algebra Tutorial:

$T: P_2(R) \to P_3(R) \quad T(f(x)) = xf(x) + f`(x)$

$U: P_3(R) \to M_{2x2}(R) \quad U\big(g(x)\big) = \begin{bmatrix} g(0) & g(1) \\ g(-1) & -g(0) \end{bmatrix}$

$\alpha = (1, x, x^2)$ basis of $P_2(R)$

$\beta = (1, x, x^2, x^3$

$\gamma = \left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right)$ is the basis of $M_{2x2}(R)$

$[T]_\alpha^\beta = ? \quad [U]_\beta^\gamma = ? \quad UT = ?$

$[UT]_\alpha^\gamma = ? \quad [UT]_\alpha^\gamma = [U]_\beta^\gamma [T]_\alpha^\beta \quad UT(x^2 + 3\text{x} + 1) = ?$

$$T(1) = x * 1 + 1` = x \to \big[T(1)_\beta\big] = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$T(x) = x * x + 1 = x^2 + 1 \to \big[T(x)_\beta\big] = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$T(x^2) = x * x^2 + 2x = x^3 + 2x \to \big[T(x^2)_\beta\big] = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

$$[T]_\alpha^\beta = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$U(1) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \to [U(1)]_\gamma \to \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

$$U(x) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \rightarrow [U(x)]_\gamma \rightarrow \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}$$

$$U(x^2) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \rightarrow [U(x^2)]_\gamma \rightarrow \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$U(x^3) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \rightarrow [U(x^3)]_\gamma \rightarrow \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}$$

$$\rightarrow [U]_\beta^\gamma = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 0 & 0 & 0 \end{bmatrix}$$

$$UT(f) = U\big(T(f)\big) = U\big(xf(x) + f`(x)\big) = \begin{bmatrix} f`(0) & f(1) + f`(1) \\ -f(-1) + f`(1) & -f`(0) \end{bmatrix}$$

$$UT(1) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$UT(x) = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

$$UT(x^2) = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$$

$$[UT]_\alpha^\gamma = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 3 \\ -1 & 2 & -3 \\ 0 & -1 & 0 \end{bmatrix}$$

$$[U]_\beta^\gamma [T]_\alpha^\beta = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 3 \\ -1 & 2 & -3 \\ 0 & -1 & 0 \end{bmatrix} = [UT]_\alpha^\gamma$$

$$\therefore \left[UT_{(x^2+3x-1)}\right] = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 3 \\ -1 & 2 & -3 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \\ 4 \\ 3 \end{bmatrix} \qquad \therefore UT_{(x^2+3x-1)} = \begin{bmatrix} 3 & 8 \\ 4 & -3 \end{bmatrix}$$