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still spans V. In particular
$|G| \geq n = |L|$.

PF from Lemma let β be a basis of V w/ $|\beta| = n$, let γ be another basis of V

Then 1. γ generates & β is lin indep. So by replacement,

$$n = |\beta| \leq |\gamma|$$

2. Take some $\gamma' \subset \gamma$ that has $|\gamma'| = n$.
Then γ' is a subset of a lin indep. s.t. γ' is lin indep. and β is generating, so by replacement, $|\gamma'| \leq |\beta| = n$.
So every finite subset of γ' has at most n elements so $|\gamma| \leq n \Rightarrow |\gamma| = n$

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observation If $\sum \alpha_i u_i = 0$.

then any vector that appears with a coeffecient not equal to 0 is a L.C. of the others.

i.e. suppose for a specific j, $\alpha_j \neq 0$

example: $0.5u_1 + 0u_2 + 7u_3 = 0$

$$0.5u_1 = -0u_2 - 7u_3$$

$$u_1 = \frac{-0u_2 - 7u_3}{0.5}$$

$$= 0u_2 - 14u_3$$

$$\alpha_j u_j + \sum_{i \neq j} \alpha_i u_i = \sum \alpha_i u_i = 0$$

$$\Rightarrow \alpha_j u_j = \sum_{i \neq j} (-\alpha_i) u_i \; / \; \alpha_j^{-1}.$$

$$\Rightarrow u_j = \sum_{i \neq j} \frac{-\alpha_i}{\alpha_j} u_i$$

Pf of replacement.

By induction on $|L|$ w/o ceremony

If $|L|=0$ nothing to prove. $(R=\phi)$

Assume lemma for sets w/ n elements

assume $|L|=n+1$

$$L=\{v_1, \dots v_{n+1}\}.$$

by $\text{span}((G\setminus R')\cup L')=V$, can write

$$v_{n+1}=\sum_{i=1}^{m} a_i u_i + b_1 v_1 + \dots + b_n v_n$$

for $u_i \in G \setminus R'$, so

$$0=\sum_{i=1}^{m} a_i u_i + b_1 v_1 + b_n v_n - v_{n+1}$$

in this lin. comb at least one of

the a_j's is non-zero.

or else, we have

$$0=a_1 v_1 + a_2 v_2 + a_n v_n - v_{n+1}$$

contracting the lin. ind of L