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<u>Example 1</u>

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \end{pmatrix} = \begin{array}{c} \begin{array}{cc} -2 & 5 \end{array} \\ \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} \\ \begin{array}{cc} -2 & 11 \end{array} \end{array}$$

$$C_{11} = \sum_{j=1}^{3} a_{1j} b_{j1} = a_{11} b_{11} + a_{12} b_{21} + a_{13} b_{31}$$

$$= 1 \cdot 1 + 2 \cdot 0 + 3 \cdot (-1) = -2$$

$$C_{12} = \sum_{j=1}^{3} a_{1j} b_{j2} = a_{11} b_{12} + a_{12} b_{22} + a_{13} b_{32}$$

$$= 1 \cdot 0 + 2 \cdot 1 + 3 \cdot 1 = 5.$$

$$C_{21} = 4 \cdot 1 + 5 \cdot 0 + 6 \cdot (-1) = -2$$

$$C_{22} = 4 + 0 + 5 + 6 = 11$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \left[\begin{pmatrix} -2 & 5 \\ -2 & 11 \end{pmatrix} \right].$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 19 & 22 \\ 43 & 50 \end{pmatrix}$$

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Example 2:

$$[R_\alpha] = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

$$[R_\beta] = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix}$$

$$\begin{pmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) \\ \sin(\alpha+\beta) & \cos(\alpha+\beta) \end{pmatrix} = [R_{\alpha+\beta}] = [R_\beta]\cdot[R_\alpha]$$

$$R_{\alpha+\beta} = R_\beta \circ R_\alpha$$

$A \in M_{m\times n}$ $\quad V \in M_{n\times 1} \cong F^n$

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$A\cdot V = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \begin{bmatrix} a_{11}x_1 + \cdots + a_{1n}x_n \\ \vdots \\ a_{m1}x_1 + \cdots + a_{mn}x_n \end{bmatrix}$$

$\Rightarrow A\cdot V \in M_{m\times n} = F^m \rightsquigarrow T_A V.$

$M_{m\times n}$: $0, I_n, +, \cdot$ not a field! (of matrix algebra)

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Good	Bad
1. $A+B=B+A$ $(A+B)+C=A+(B+C)$ $A+0=A$ $A+(-A)=0$	1. addition defined only for matrices of the same size $+: M_{m\times n} \times M_{m\times n} \to M_{m\times n}$
2. $A(BC)=(AB)C$ $A\in M_{m\times n}$ $\exists I_n$ s.t. $A I_n = A$ $\exists I$ s.t. $I_m A = A$ $m\times n$ $I_n = \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}$	2. mult make sense iff the "middle" dim is the same $M_{m\times n} \times M_{n\times p} \to M_{m\times p}$
3. If $A^{-1}A=I$ then "A^{-1} is unique" & $AA^{-1}=I$ $A\in M_{n\times n}$ $L: V \underset{L^{-1}}{\overset{L}{\rightleftarrows}} W$	3. even if $A\in M_{n\times n}$ & $A\neq 0$ A^{-1} may not exist $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $D: P_3 \to P_3$ $D^{-1} \nexists$.
4. $(A+B)C=AC+BC$ (when possible) $A(B+C)=AB+AC$ Proofs 1. use the explicit fornulas for matrix mult 2. First prove it for l.t. $(T+S)\circ Z = T\circ Z + S\circ Z$	4. Generally $A\cdot B \neq B\cdot A$ $A\in M_{3\times 2}$ $B\in M_{2\times 4}$.