

Sep 18, 2012

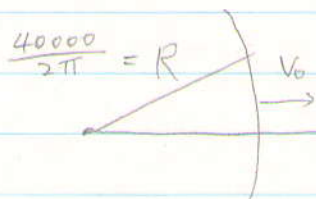
Mat 267

1/2

$$m(x)dx + n(y)dy = 0 \Rightarrow M(x) + N(y) = C$$

"separated" $\leftrightarrow m + ny' = 0$

example 2 (escape velocities)



$$F = -\frac{mgR^2}{(R+x)^2} \Rightarrow \frac{dv}{dt} = -\frac{gR^2}{(R+x)^2}$$

$$\Rightarrow v \frac{dv}{dx} = -\frac{gR^2}{(R+x)^2} \Rightarrow \frac{v^2}{2} = \frac{gR^2}{R+x} + C$$

$$v(0) = V_0$$

$$\text{At } x=0 \quad \frac{V_0^2}{2} = gR + C \Rightarrow C = \frac{V_0^2}{2} - gR$$

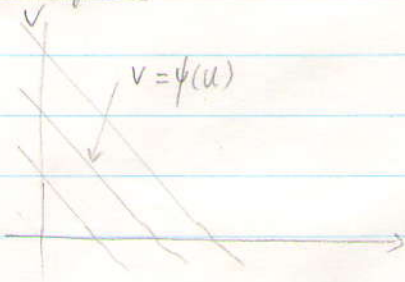
$$\frac{v^2}{2} = \frac{gR^2}{R+x} + \frac{V_0^2}{2} - gR$$

For which V_0 , $\lim_{x \rightarrow \infty} v(x) = 0$?

$$\Rightarrow 0 = 0 + \frac{V_0^2}{2} - gR$$

$$\Rightarrow V_0 = \sqrt{2gR}$$

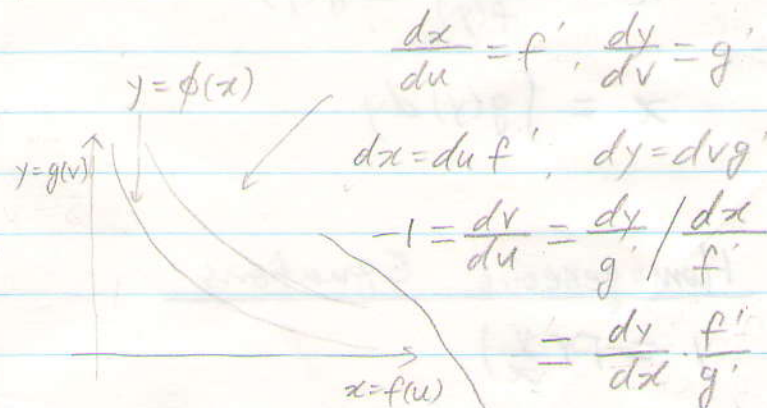
example



$$u + v = C$$

$$\frac{dv}{du} = -1$$

$$x = e^u \\ y = e^v$$



$$\frac{dx}{du} = f', \quad \frac{dy}{dv} = g'$$

$$dx = du f', \quad dy = dv g'$$

$$-1 = \frac{dv}{du} = \frac{dy}{dx} \cdot \frac{dx}{du} = \frac{dy}{dx} \cdot \frac{f'}{g'}$$

$$= \frac{dy}{dx} \cdot \frac{f'}{g'}$$

$$\frac{dy}{dx} = -\frac{g'}{f'} \Rightarrow \frac{dy}{g'} + \frac{dx}{f'} = 0$$

$$v = \psi(u)$$

$$g^{-1}(y) = \psi(f^{-1}(x))$$

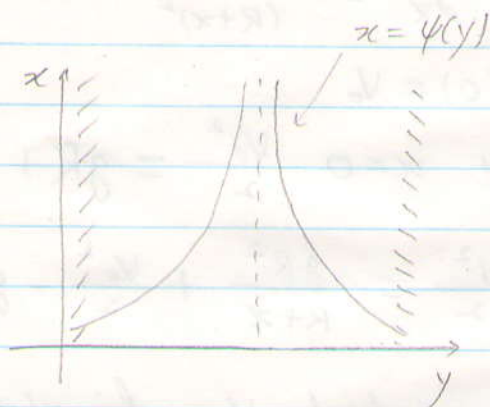
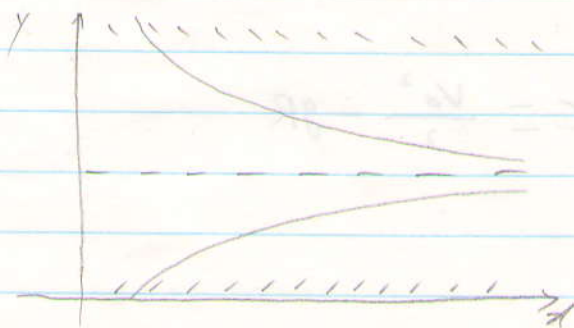
$$y = g(\psi(f^{-1}(x)))$$

$$\phi = g \circ \psi \circ f^{-1}$$

Autonomous Equations

$$y' = f(y) \quad \frac{dy}{dx} = f(y) \quad \frac{dy}{f(y)} = dx$$

$$\frac{y'}{f} - 1 = 0$$



$$x' = g(y)$$

$$\frac{dy}{dx} = f(y) \quad \frac{dx}{dy} = \frac{1}{f(y)}$$

$$x' = \frac{1}{f(y)} = g(y)$$

$$x = \int g(y) dy$$

Homogeneous Equations

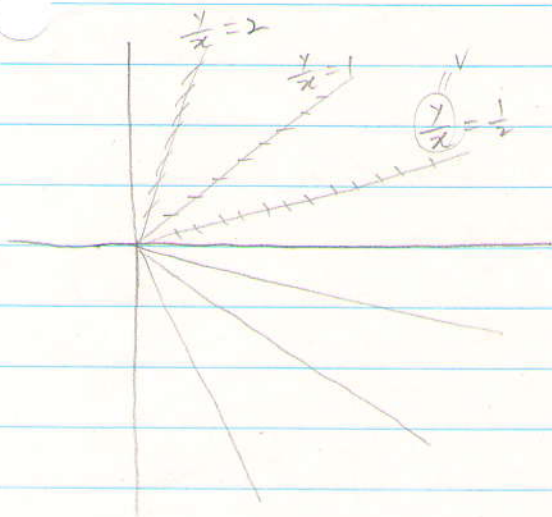
$$y' = F\left(\frac{y}{x}\right)$$

$$y' = \frac{y^2 + 2xy}{x^2} = \frac{y^2/x^2 + 2\frac{y}{x}}{1} = \left(\frac{y}{x}\right)^2 + 2\frac{y}{x}$$

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2/2



$$1 + v \text{ gal} - v \text{ gal} = 2 + 1 \text{ gal} \leftarrow$$

$$1 + v \text{ gal} =$$

$$xv = v \leftarrow \frac{v}{1+v} = x \leftarrow$$

$$xv = v \leftarrow$$

Solution

$$v = \frac{y}{x} \Rightarrow y = v \cdot x$$

$$y' = v'x + v$$

$$y' = F\left(\frac{y}{x}\right) \Rightarrow v'x + v = F(v)$$

$$xv' = F(v) - v$$

$$x \frac{dv}{dx} = F(v) - v$$

$$\frac{dv}{F(v) - v} = \frac{dx}{x}$$

$$y' = \left(\frac{y}{x}\right)^2 + 2 \cdot \frac{y}{x} \quad y = x \cdot v$$

$$xv' + v = v^2 + 2v$$

$$xv' = v^2 + v = v(v+1)$$

$$\frac{dv}{v(v+1)} = \frac{dx}{x}$$

$$A(v+1) + Bv = 1 \Rightarrow A=1, B=-1$$

$$\log|x| + C = \int \frac{dv}{v(v+1)} = \int \left(\frac{A}{v} + \frac{B}{v+1} \right) dv$$

$$\log|x| + C = \int \left(\frac{1}{v} - \frac{1}{v+1} \right) dv$$

$$\Rightarrow \log|x| + C = \log|v| - \log|v+1|$$

$$= \log \left| \frac{v}{v+1} \right|$$

$$\Rightarrow C \cdot x = \frac{v}{v+1} \Rightarrow v = \frac{-Cx}{Cx-1}$$

$$\Rightarrow y = \frac{-Cx^2}{Cx-1}$$



reduktion

$$xv = y \Leftrightarrow \frac{y}{x} = v$$

$$y = vx + v = v(x+1)$$

$$(v) \cdot (x+1) = y \Rightarrow \left(\frac{y}{x+1}\right) \cdot (x+1) = y$$

$$xv = y \Rightarrow vx = y$$

$$v = \frac{y}{x} = \frac{y}{x+1} \cdot (x+1)$$

$$\frac{y}{x} = \frac{y}{x+1} \cdot (x+1)$$

$$y = \frac{y}{x} \cdot (x+1) = y \cdot \left(\frac{x+1}{x}\right)$$

$$xv + v = vx + v$$

$$(1+v)v = vx + v = vx + v$$

$$\frac{y}{x} = \frac{y}{x+1} \cdot (x+1)$$

$$\log \left| \frac{y}{x+1} \right| = \log \left| \frac{y}{x} \right| = \log \left| \frac{y}{x} \right|$$

$$\log|x| + C = \log|y| - \log|x+1|$$