Def^n

$\mathbb{C} = \{(a,b) : a,b \in \mathbb{R}\}$

Turn this into a field candidate

Def^{ns} $0_{\mathbb{C}} := (0,0)$, $1_{\mathbb{C}} := (1,0)$

$(a,b) + (c,d) := (a+c, b+d)$, $(a,b)\cdot(c,d) := (ac-bd, bc+ad)$

$-(a,b) := (-a,-b)$, $(a,b)^{-1} := \left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$ only if $(a,b) \neq 0_{\mathbb{C}}$ ie $0,0 \Rightarrow 0 \neq 0 \wedge b \neq 0$

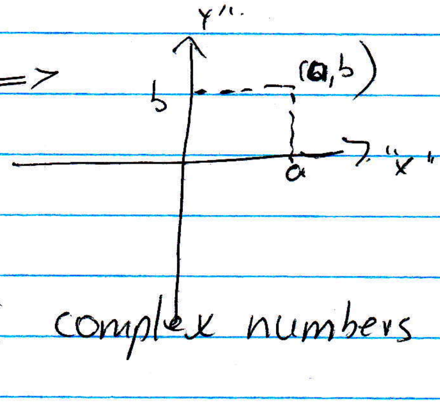
$i := (0,1)$, $z : \mathbb{R} \xrightarrow{\text{(function)}} \mathbb{C}$: eg $7 \longmapsto (7,0)$

Theorem : $\mathbb{C}$ is a field, $i^2 = -1$, $z : \mathbb{R} \to \mathbb{C}$ has the two desired properties ($a,b \in \mathbb{R}$, $z(a\cdot b) = z(a)\cdot z(b)$, $z(a+b) = z(a) + z(b)$).

eg

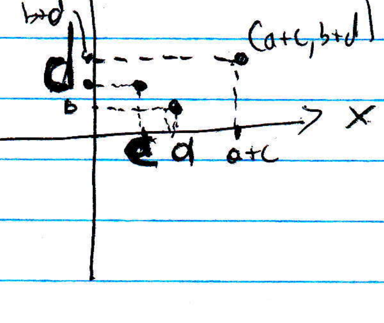
$$3 + 7i = (3,0) + (7,0)(0,1)$$
$$= (3,0) + (7\cdot 0 - 0, 1 + 0\cdot 0)$$
$$= (3,0) + (0,7) = (3,7)$$

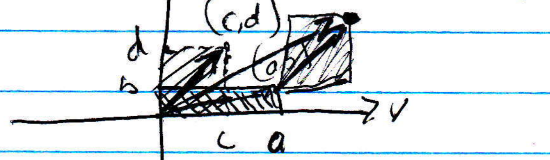
$a + bi \longleftrightarrow (a,b) \Longleftrightarrow$



(can think of complex numbers as points on a plane.

$(a,b) + (c,d) = (a+c, b+d)$





(added both vectors) (moved one rectangle to the top right of the other.