Lecture # 4
21/09/06

Force Vectors

Force has a direction and a magnitude

Special force vector called 0.

1 They can be added $x+y$
2 They can be multiplied by any scalar (number)

Props $\forall x,y \in V$ ① $x+y=y+x$ — x is used for vectors. a,b are used for scalars

$\forall x,y,z \in V$ ② $x+(y+z)=(x+y)+z$

$\exists 0 \in V$ s.t $\forall x$ ③ $x+0=x$

④ $\forall x \; \exists y$ s.t $x+y=0$ — Conversion

⑤ $1 \cdot x = x$ — x,y,z — vector

⑥ $a(bx)=(a \cdot b)x$ — a,b,c — scalar

⑦ $a(x+y)=ax+ay$

$\forall x \in V \; \forall a,b \in F$ ⑧ $(a+b)x = ax+bx$

Def: Let F be a "field" ("of scalars"). A vector space over F is a set V (of "vectors") along with two operations $+: V \times V \longrightarrow V$ (binary operation)

$\cdot : F \times V \longrightarrow V$

$\{horse, cow\} = \{cow, horse\}$ order doesn't matter $\{\}$

$(pig, monkey) \neq (monkey, pig)$ order does matter $()$

For Force vectors

$x \to |x|$ 4. $|x+y| \leq |x| + |y|$ (Don't need this to define vector spaces.

In $\mathbb{Q}^3$ $\left(\frac{3}{2}, -2, 7\right) + \left(-\frac{3}{2}, \frac{1}{3}, 240\right) = \left(0, -\frac{5}{3}, 247\right)$

2 $V = M_{m \times n}(F) = \left\{ \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}_{m\times n} : a_{ij} \in F \right\}$ $M_{3\times 2}(\mathbb{R}) \ni \begin{pmatrix} 7 & -7 \\ \pi & e \\ -5 & 2 \end{pmatrix}$

$M_{2\times 2}$ $\begin{pmatrix} a_{11} & a_{11} \\ a_{21} & a_{22} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{pmatrix}$

Multiply by a is mult. of all by a.

3. $\mathbb{C}$ form a V.S. (vector space) over $\mathbb{R}$.

4. F is a V.S. over itself
5. $\mathbb{R}$ is a V.S. over $\mathbb{Q}$.
6. let S be a set. Let $\mathcal{F}(S, R) = \{ f : S \to R \}$

* each number assinged to each student is a function.
* collection all the functions is a vector space

$f, g \in \mathcal{F}(S, \mathbb{R})$

$(f+g)(t) = f(t) + g(t)$

999001 → student no
1.78 → height