and then $\alpha(A) = \beta(B)$, and $V(Y,\alpha) = V(Y,\beta)$.

pf. $V(Y,\beta) = \int_B V(D\beta) = \int_B V(D(\alpha\circ g)) = \int_B V((D\alpha)\circ g \cdot Dg) = \int_B |\det((Dg)^T \cdot ((D\alpha)\circ g)^T \cdot (D\alpha)\circ g \cdot Dg)|^{1/2}$

$= \int_B |\det Dg| \, (V(D\alpha)\circ g) = \int_A V(D\alpha) = V(Y,\alpha)$.

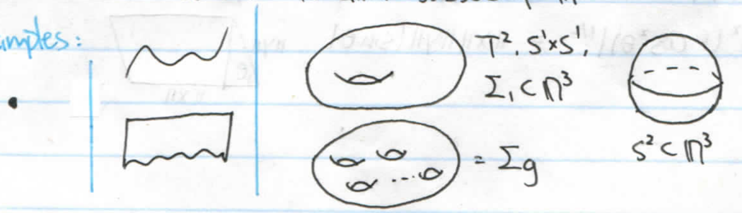
Jan 18

Def. $\int_Y f\,dV = \int_A (f\circ\alpha)\,V(D\alpha) \overset{\text{thm}}{=} \int_B (f\circ\beta)\,V(D\beta)$

M and $\partial M \leftarrow$ boundary of M.

M: " A nice & smooth k-dim subset of $\mathbb{R}^n$ "

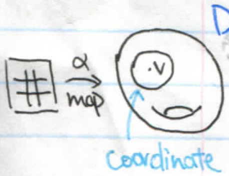
Examples:



K-bottle $\subset \mathbb{R}^4$

$O(n) = \{A \in M_{n\times n} : A^T A = I\} \subset \mathbb{R}^{n^2}$

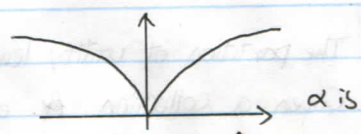
$O(2) = S^1 \cup S^1 \subset \mathbb{R}^4$



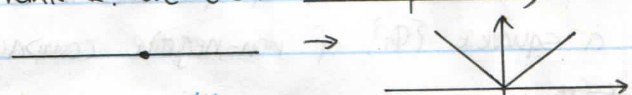
Def. A k-dim manifold (w/o boundary, of class C^r, $r \geq 1$) in $\mathbb{R}^n$, is a subset $M \subset \mathbb{R}^n$ s.t. each $p \in M$ has an open nbd V s.t. there is an open $U \subset \mathbb{R}^k$ & a C^r homomorphism $\alpha: U \to V$ whose differential $D\alpha(x)$ has rank for every $x \in U$.

non-examples: 1. $\alpha: \mathbb{R} \to \mathbb{R}^2$ by $t \mapsto (t^3, t^2)$

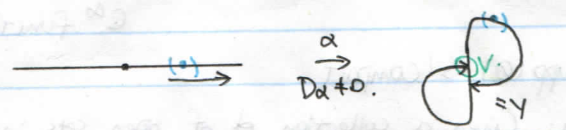
$D\alpha = (3t^2, 2t)$ not rank 1. at $t=0$.



2. $t \mapsto (t^3, |t|^3)$.



3.



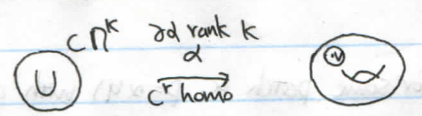
V contains vertical lines & $(x\to\infty)$.

not manifold.

∂M on friday.

Jan 20.

$\int_{[a,b]} f' = f|_b^a$.



manifold with boundary:

Premitive. Def. Let $H^k = \{x \in \mathbb{R}^k, x_k \geq 0\}$ A k-manifold (of class C^r, possibly with boundary) is $M \subset \mathbb{R}^n$ s.t. each $p \in M$ has an open nbd (in M) s.t. there is an open $U \subset H^k$ & a C^r homemorphism $\alpha: U \to V$ whose differential is of rank k at every $x \in U$.

The ∂M ($\neq$ BdM.) of M is $\partial M = \{p \in M : \text{for some patch } \alpha.\ p = \alpha(q) \text{ where } q \in \partial H^k\}$. ($\partial H^k = \mathbb{R}^{k-1}\times\{0\}$)

Claim: ∂M is also a manifold, without boundary, of dim $k-1$.

Issues: 1. What does differentiability mean on H^k?

2. Could it be the case that $\partial M = M$?