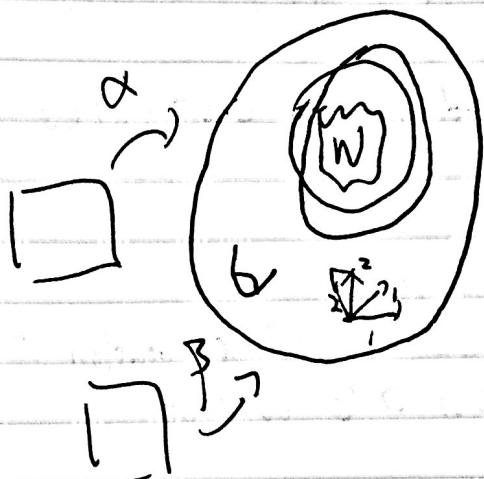


20th Mon March Hour 065

Read along: 33-35, 37 skip 36.



$$\int_M^{\alpha} f \, d\omega = \pm \int_M^{\beta} f \, d\omega$$

Def: Orientation on M is a cont/consistent choice of a pos-det class of ordered bases of $T_x M$, for each $x \in M$

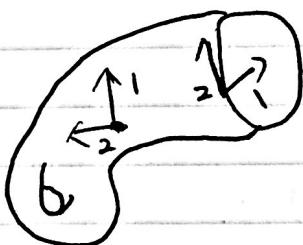
Vocab: "orientable", "oriented"

Claim: If M is connected, it has 0 or 2 orientations

Claim: If $M \subset \mathbb{R}^{k+1}$, orientation \iff "sides" (choose extra normal)

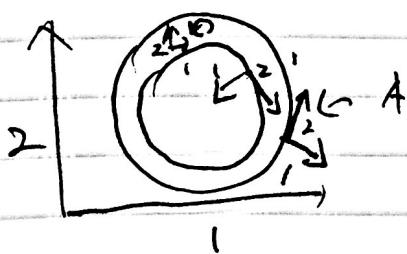
Claim: If M is oriented, then ∂M is ∂M . by " ∂M 's

orientation is such that prepending to it, the outward normal of ∂M gives M 's orientation."



Example: 1. $A = \{x \in \mathbb{R}^2 : 1 \leq \|x\| \leq 2\}$, oriented by the orientation inherited from the std orientation of \mathbb{R}^2 (e_1, e_2)

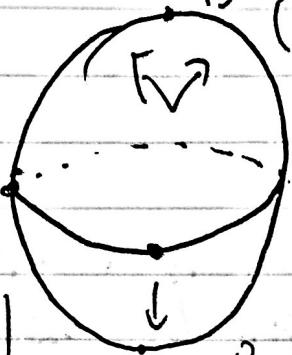
∂A : oriented with outer boundary counter-clockwise, and inner boundary clockwise



Example: $D^3 = \{X \in \mathbb{R}^3 : \|X\| = 1\} \subset \mathbb{R}^3$
oriented standardly, e_1, e_2, e_3

$\partial D^3 = S^2$. Q: How is S^2 oriented at $P = (1, 0, 0)$

$$P_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$



$$P = (1, 0, 0)$$

$$q_S = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

$$P_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

How is $T_p S^2$ oriented? $e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$



is it (e_3, e_2) ?

(e_1, e_3, e_2) is std orientation of \mathbb{R}^3
 $\Rightarrow (e_2, e_3)$ is correct.

$(-e_1, e_3, e_2)$ is correct.

$$-1 + \text{C} \curvearrowright$$

Exercise: $T_{P_2} S^2$ is oriented by (e_3, e_1)

$$T_{P_3} S^2 \xrightarrow{\hspace{1cm}} (e_1, e_2)$$

If: $T: V \rightarrow W$ is an isomorphism of oriented v.s.

Then: T is either "orientation preserving":

\Leftrightarrow pushes a basis agreeing with orientation of V to
a basis agreeing with orientation of W .

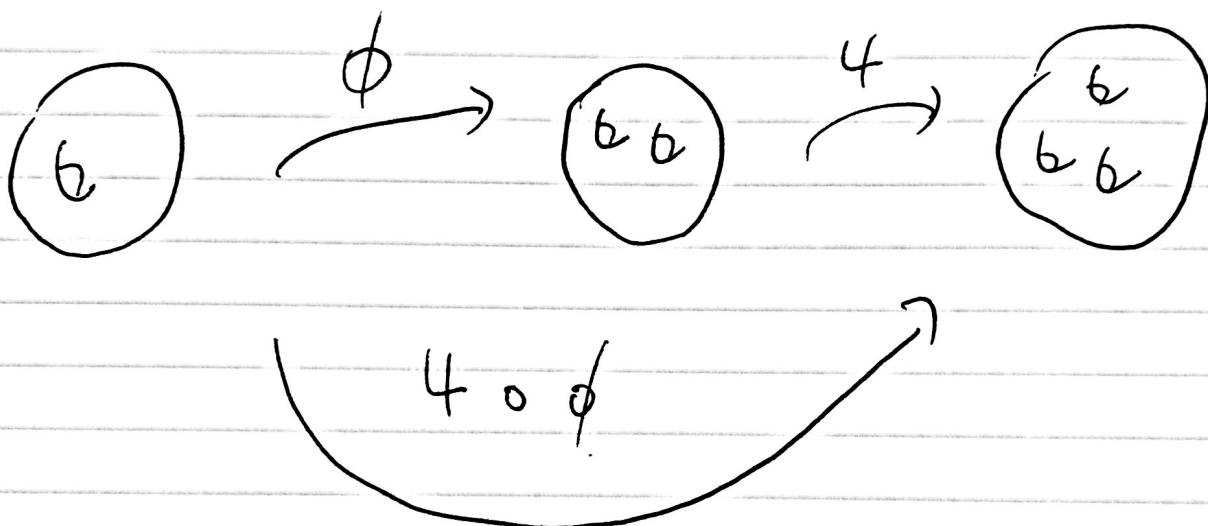
or "orientation reversing reversing"
 \Leftrightarrow otherwise

If $\phi: M \rightarrow N$ is a c^r map, whose differential is of maximal rank.
 Then it can be "orientation preserving" or "positive"
 $\Leftrightarrow D\phi_p : T_p M \rightarrow T_{\phi(p)} N$ is orientation preserving

or "orientation reversing"

$$\Leftrightarrow \sim$$

The composition of two positive maps is positive



Integration only happens on oriented manifolds.

We only use positive charts. In that case.

$$\int_M w = \int_M^P w$$

Pf: go back a few classes, and $g = P^{-1} \circ \alpha$

$\uparrow \quad \uparrow$

positive positive