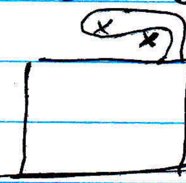
Sept 17/14

MAT 240 LECTURE #4

READ ALONG: APPENDICES A-D, SEC 1.1 & 1.2

RIDDLE ALONG: CAN YOU HANG A PICTURE ON TWO NAILS, SO THAT IF YOU REMOVE ONE OF THEM THE PICTURE WILL FALL. (WHAT ABOUT 3 NAILS?)



Def$^{\underline{n}}$: The "characteristic" of a field, F, is the least possible integer n s.t. $\iota(n)=0$ if such an integer exists, otherwise it is 0. It is denoted $char(F)$.

Ex $Char(\mathbb{R})$ ι must map $\mathbb{Z}$ to itself $\in \mathbb{R}$.

$\Rightarrow Char(\mathbb{R})=0$ (b/c no positive integer $=0$)

$Char(\mathbb{Q})=0$

$Char(F_2)=2$ $\quad(\iota(1)=1\neq 0,\ \iota(2)=1+1=0_F)$

(There are fields whose char. is also 2)

$Char(F_6)=7$ $\quad(\text{i.e., } \iota(3)=3,\ \iota(6)=6,\ \iota(7)=0)$

Claim: The Char. of a field F is either 0 or a prime #.

Proof: By Contradiction.

Suppose $char(F)=m\cdot n$. (where $m,n>1$, & $m,n<char(F)$).

Then $\iota(m\cdot n)=0=\iota(m)\cdot\iota(n)$ $\therefore$ $\iota(m)=0$ or $\iota(n)=0$

If $\iota(m)=0$ then we found a positive integer m which is smaller than $Char(F)$ s.t. ι of that integer is 0.

This contradicts of the $Char(F)$. Likewise for $\iota(n)=0$ (reversing).

In reality the notation "z" is never used.
In practice people write 7 for z(7) or maybe 7_F.

The Complex Numbers

$+, \cdot, -, \div, \sqrt{\ }$

Theorem.
There is a field $\mathbb{C}$ that contains the ordinary $\mathbb{R}$, and in addition has an element called i, s.t. $i^2 = -1$.

[Dream, Implementations, Formalization, Proof]

Dream: "i exists in some field" ($\mathbb{C}$ exists)

b/c $\times$ in field — b/c sum (+) in field — should be able to apply distribution law

$\mathbb{C} = \{\ldots 7, i, 0, 1, 7i, 3, 3+7i\}$

$$(a+bi)(c+di)$$
$$= a(c+di) + bi(c+di)$$
$$= ac + adi + bic + bidi$$
$$= (ac + bdi^2) + (ad + bc)i$$
$$= \underbrace{(ac - bd)}_{\mathbb{R}} + \underbrace{(ad + bc)}_{\mathbb{R}} i$$

$$(a+bi)^{-1} = \frac{1}{(a+bi)} \cdot \frac{(a-bi)}{\underbrace{(a-bi)}_{\text{"conjugate of } a+bi\text{"}}} = \frac{a-bi}{a^2+b^2 + \cancel{(a)(-b)i} + \cancel{b(a)i}}$$

$\neq$

$$i_{\circ\circ} = \frac{a-bi}{a^2+b^2}$$

$$= \underbrace{\frac{a}{a^2+b^2}}_{\mathbb{R}} + \underbrace{\left(\frac{-b}{a^2+b^2}\right)}_{\mathbb{R}} i$$

(If $a^2+b^2=0$, then $a+bi=0$ then would not need to be able to invert, the inverse of the form $i_{\circ\circ}$ is of the form previously considered)