1. We can structct a Seqeence

$C_N = [\frac{1}{N}, \sqrt{2\lambda} - \frac{1}{N}] \nearrow (0, \sqrt{2\lambda})$ for $\lambda > 0$

and $C_N' = [\sqrt{-2\lambda} + \frac{1}{N}, -\frac{1}{N}] \nearrow (\sqrt{-2\lambda}, 0)$ for $\lambda < 0$

for C_N : $\frac{1}{N+1} < \frac{1}{N}$ $\sqrt{2\lambda} - \frac{1}{N+1} > \sqrt{2\lambda} - \frac{1}{N}$

So $C_N \subset \text{Int } C_{N+1}$

for C_N' : $-\frac{1}{N} < -\frac{1}{N+1}$ $\sqrt{-2\lambda} + \frac{1}{N+1} < \sqrt{-2\lambda} + \frac{1}{N}$

So $C_N' \subset \text{Int } C_{N+1}'$

Since it is in $\mathbb{R}$, each C_N is close and bounded, they are compact.

each C_N has bd of two points which is measure of zero.

So they are rectifiable.

$\frac{2\sqrt{2\lambda}}{N}$

$\lim_{N\to\infty} \int_{\frac{1}{N}}^{\sqrt{2\lambda}-\frac{1}{N}} x\,dx = \lim_{n\to\infty} \frac{(\sqrt{2\lambda}-\frac{1}{N})^2}{2} - \frac{(\frac{1}{N})^2}{2} = \lambda$

$\lim_{N\to\infty} \int_{\sqrt{-2\lambda}+\frac{1}{N}}^{-\frac{1}{N}} x\,dx = \lim_{n\to\infty} \frac{(-\frac{1}{N})^2}{2} - \frac{(\sqrt{-2\lambda}+\frac{1}{N})^2}{2} = \lambda$

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$C_N = [-n, n]$

$\int_{C_N} |f|$ not is bounded. $\lim_{n\to\infty} [-n, n] \uparrow \mathbb{R}$

$\int_{C_N} |x| = \cancel{\int_{-N}^{0} -x\,dx + \int_0^{\frac{1}{N}} x\,dx}$

$\int_{-N}^{0} -x\,dx + \int_0^N x\,dx$

$= 0 + \frac{N^2}{2} + \frac{N^2}{2} - 0 = N^2$ not bounded.

So ~~$\nexists_{\mathbb{R}} f$ exists and equals~~

$\cancel{= \lim_{N\to\infty} \int_{-N}^{N} x\,dx = \lim_{n\to\infty} \frac{N^2}{2} - \frac{(-N)^2}{2} = 0}$

So $\int f$ does not exist.

3.a) $(1, n) \times (1, n) \nearrow A$ $\quad n > 1$

since x, y of A are positive

$|f| = f$

$$\int_{U_N} |f| = \int_1^n \int_1^n \frac{1}{\sqrt{xy}}\,dx\,dy$$

$$= \int_1^n \frac{2\sqrt{x}}{\sqrt{y}}\Big|_1^n\,dy = \int_1^n \frac{2\sqrt{n}-2}{\sqrt{y}}\,dy$$

$$= 4(\sqrt{n}-1)\sqrt{y}\Big|_1^n\,dy$$

$$= 4(\sqrt{n}-1)^2 \quad \text{not bounded.}$$

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So $\int_A f$ doesn't exist

$(1/n, 1) \times (1/n, 1) \nearrow B$ $\quad n > 0$.

$|f| = f$ ~~to~~. as well

$$\int_{U_N} |f| = \int_{\frac{1}{n}}^1 \int_{\frac{1}{n}}^1 \frac{1}{\sqrt{xy}}\,dx\,dy$$

$$= \int_{\frac{1}{n}}^1 \frac{2\sqrt{x}}{\sqrt{y}}\Big|_{\frac{1}{n}}^1\,dy = \int_{\frac{1}{n}}^1 \frac{2-2\sqrt{\frac{1}{n}}}{\sqrt{y}}\,dy$$

$$= 4\left(1-\sqrt{\tfrac{1}{n}}\right)\sqrt{y}\Big|_{\frac{1}{n}}^1 = 4\left(1-\sqrt{\tfrac{1}{n}}\right)^2$$

Since ~~it is~~ b $n > 1$.

sequence is bounded by 0 and 4.

since $0 < 1-\sqrt{\frac{1}{n}} < 1$

$$\text{So } \int_B f = \lim_{n\to\infty} \int_{U_N} f$$

$$= \lim_{n\to\infty} 4\left(1-\sqrt{\tfrac{1}{n}}\right)^2 = 4.$$

6. $(1, n] \nearrow x$

~~$(\frac{1}{n}, 1) \nearrow (0, \frac{1}{x}) \nearrow y$~~

Since x, y all positive

$|f| = f$.

~~$(1, n) \times (0, n)$ is rectifiable~~

$$\int_1^n \int_0^{\frac{1}{x}} \frac{1}{x\sqrt{y}}\, dy\, dx$$

the set of $\{(1, n) \ni x,\ y \in (0, \frac{1}{x})\}$ is bounded and ~~contin~~ disco set is measure of zero so integral exists.

$$= \int_1^n \frac{2\sqrt{y}}{x}\Big|_0^{\frac{1}{x}} dx$$

$$= \int_1^n \frac{2}{x^{\frac{3}{2}}}\, dx$$

$$= -2\,\frac{2}{\sqrt{x}}\Big|_1^n = \frac{-4}{\sqrt{n}} + 4.$$

since $n > 1$.

$\int_{U_N} |f|$ ~~$\int_{U_N} |f|$~~ is bounded by 4 and 0.

$\int_A f$ exists

$$= \lim_{n\to\infty} \int_{U_N} f = \lim_{n\to\infty} \frac{-4}{\sqrt{n}} + 4 = 4$$