$u \in \mathrm{span}(S_2)$ eg. $u = 2v_1 - v_2$

$u = 2(7u_1 + 3u_2) - (2u_1 - u_2)$

$= 12u_1 + 7u_2$

Cont. w/ aside

3. If S_1 spans V & $S_1 \subset \mathrm{span}(S_2)$, then S_2 spans V

Indeed, by 2,

$V = \mathrm{span}(S_1) \subset \mathrm{span}(S_2) \subset V$

$\Rightarrow \mathrm{span}(S_2) = V \Rightarrow S_2$ spans V.

To show $\mathrm{span}(S_2) = M_{2\times 2}(F_5)$ by 3

it is enough to show that $M_1 \in \mathrm{span}(S_2)$

$M_2 \in \mathrm{span}(S_2)$ ...

indeed $\frac{1}{3}(N_1 + N_2 + N_3 + N_4 - 3N_1) = M_1$

Def A subset $S \subset V$ is "linearly dependent" ("bad", "wasteful") if $\exists \alpha_i \in F$ not all equal to 0, & distinct $u_i \in S$ s.t.

$$0 = \sum \alpha_i u_i$$

otherwise, we say that S is "linear independent"

example 1

$$S = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}, \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix} \right\} \subset \mathbb{R}^3$$

S is lin. dep. Indeed

$$-u_1 + 2u_2 - u_3 = 0.$$

example 2

In F^n, $\left\{ u_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, u_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \cdots u_n \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} \right\}$

is lin indep. Indeed if $\sum \alpha_i u_i = 0$, then

$$\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} = 0_V = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix} \Rightarrow (\alpha_1 = 0) \wedge (\alpha_2 = 0) \cdots \wedge (\alpha_n = 0)$$

So this isn't a non-trivial combination