

Munkres, section 2

4. (a) Let  $A$  be an  $n$  by  $n$  matrix of rank  $n$ . By applying elementary row operations to  $A$ , one can reduce  $A$  to the identity matrix. Show that by applying the same operations, in the same order, to  $I_n$ , one obtains the matrix  $A^{-1}$ .
- (b) Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}.$$

Calculate  $A^{-1}$  by using the algorithm suggested in (a). [*Hint:* An easy way to do this is to reduce the 3 by 6 matrix  $[A \ I_3]$  to reduced echelon form.]

- (c) Calculate  $A^{-1}$  using the formula involving determinants.

Munkres, section 3

3. Let  $A \subset X$ . Show that if  $C$  is a closed set of  $X$  and  $C$  contains  $A$ , then  $C$  contains  $\bar{A}$ .
4. (a) Show that if  $Q$  is a rectangle, then  $Q$  equals the closure of  $\text{Int } Q$ .
- (b) If  $D$  is a closed set, what is the relation in general between the set  $D$  and the closure of  $\text{Int } D$ ?
- (c) If  $U$  is an open set, what is the relation in general between the set  $U$  and the interior of  $\bar{U}$ ?
5. Let  $f: X \rightarrow Y$ . Show that  $f$  is continuous if and only if for each  $x \in X$  there is a neighborhood  $U$  of  $x$  such that  $f|U$  is continuous.