

wiki: ① all page titles must start with 06-240.

② if typing something, write in wiki

③ Navigation panel should remain clean & neat

④ avoid microsoft.

⑤ chats go in discussion page

Lemma 1  $a \cdot 0 = 0$  (in fact,  $a \cdot b = 0 \iff a = 0$  or  $b = 0$ )

\* or: or both

iff

← shown below.

Lemma 2  $(-a)b = -(ab)$

Proof of 1: we know that we have to use F5 because properties that define 0 belong to additive world but we are trying to prove a multiplicative statement. F5 is only axiom that uses additive & multiplicative.

$0 + 0 = 0$  (by F3)

mult by a  $a(0 + 0) = a \cdot 0$

$a0 + a0 = a0$  (by F5)

$\times 2a0 = a0 \implies$  get stuck b/c never talked about 2.

we want to

show that

$a0 \in F$  so by F4 there is an  $x$  s.t.  $a0 + x = 0$ .

So, take  $x$  and add it to both sides of the eqn.

$\times a0 + a0 + x = a0 + x \implies$  need to put brackets when adding 3's at this low level.

$(a0 + a0) + x = a0 + x$

$a0 + (a0 + x) = a0 + x$

$a0 + 0 = 0$

$a0 = 0$

by F2:  $0 + 0 = 0$

by choice of  $x(0,0) = 0$

by F3:  $0 + 1 = 0$

$(0,1) = 0$

QED/□.

Proof of Lemma 2.  $(-a)b = -(ab)$ .

Need to show that:

$$(-a) \cdot b + ab = 0$$

↳ negative of  $a \cdot b$   
which is: ~~the~~ the number  
which when you add  $ab$  to  
it will give you 0.

$$(-a) \cdot b + ab \stackrel{\text{FS}}{=} (-a + a) \cdot b$$

by def of  $-a$

$$0 \cdot b$$

by lemma 1

$$0.$$

QED.

## Complex Numbers.

$$\sqrt{-7} = \sqrt{(-1)7} = \sqrt{-1} \cdot \sqrt{7}$$

Enough to understand  $\sqrt{-1} = i : i^2 = -1$

→  $7 + 3i$  also I need to invent (need to add all conceivable #s)

$$\left( \frac{1}{2i} + \frac{-2i}{5-i} \right) \frac{5}{7+i}$$

But really, stops here. When you take 1 real # + another real #  $\times i$ .

$$\mathbb{C} = \{a + bi : a, b \in \mathbb{R}\}$$

(a, b)

↳ really, complex # is a pair of real #s.

↳ can't break up  $a + bi \Rightarrow$  it's one symbol, & + is not really +. (notation).

$$0_{\mathbb{C}} = 0 + 0i \Rightarrow \text{defined this way.}$$

$$= (0, 0)$$

$$1_{\mathbb{C}} = 1 + 0i$$

$$= (1, 0)$$

MAT 240

$(a, b) + (c, d)$

[in my imagination,

$(a, b) = a + bi$

$(c, d) = c + di$

$(a + bi) + (c + di) = (a + c) + (b + d)i$   
 $= (a + c, b + d)$

↳ notation for complex #'s ]

this is not proof: just motivation for how we chose to define it.

$= (a + c, b + d)$

Formal Def'n.

$(a, b) \cdot (c, d) = (ac - bd, bc + ad)$

[in my dreams,

$(a + bi) \cdot (c + di) = a \cdot c + a \cdot di + bi \cdot c + bi \cdot di$   
 $= a \cdot c - bd + bci + adi$   
 $= (ac - bd) + (bc + ad)i$

$= bdi^2 = -bd$

Now, we have to check F4 through F5 to see that complex #'s are ~~a~~ real Field.

F2. Associativity:

$((a, b) + (c, d)) + (e, f) \stackrel{?}{=} (a, b) + ((c, d) + (e, f))$   
 $(a + c, b + d) + (e, f) \stackrel{?}{=} (a, b) + (c + e, d + f)$   
 $((a + c) + e, (b + d) + f) \stackrel{?}{=} (a + (c + e), b + (d + f))$

just replace w/ def'n.

Is it true that

look at elts.

$(a + c) + e = a + (c + e)$  and  $(b + d) + f = b + (d + f)$ ?

Independently.

Yes, by F2 for  $\mathbb{R}$ .

F1, F2, F3, F5 are easy.

F4 is hard bc have to find a ~~b~~ given a.

Claim.

1.  $(a, b) + (-a, -b) = (0, 0) = 0_e$  (hence,  $(-a, -b)$  is a negative of  $(a, b)$ )

2. If  $(a, b) \neq 0_e$  then  
 $(a, b) \cdot \left( \frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2} \right) = (1, 0) = 1_e$ .

$$(a+bi)^{-1} = \frac{a}{a^2+b^2} - \frac{b}{a^2+b^2} i$$

---

P. 558

Proof (a):  $\overline{\overline{z}} = z$  Let  $z = a+bi$  and  $w = c+di$ , where  $a, b, c, d \in \mathbb{R}$ .

$$\overline{\overline{z}} = \overline{a+bi} = \overline{a-bi} = a - (-bi) = a+bi$$

Proof (d):  $\overline{\left(\frac{z}{w}\right)} = \frac{\overline{z}}{\overline{w}}$  if  $w \neq 0$ .

$$\overline{\left(\frac{z}{w}\right)} = \overline{\left(\frac{a+bi}{c+di}\right)} =$$

Proof (e):  $z$  is a real number if and only if  $\overline{z} = z$ .

i).  $z$  is a real number if  $\overline{z} = z$ .

$$\overline{z} = z$$

$$a+bi = a+bi$$

$$a-bi = a+bi$$

By Cancellation law a),  $-bi = bi \Rightarrow b=0$ .

$$a-0i = a+0i$$

$$a = a$$

$a$  is a real number, which  $= z$ .  $\square$

ii) If  $z$  is a real number, then there can be no imaginary part.

$$\text{So } z = a+0i = a-0i = \overline{z} \quad \square$$