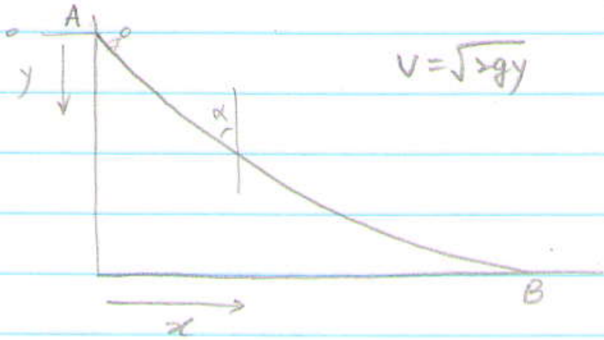
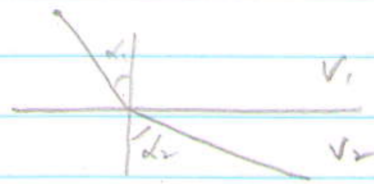


$V = \sqrt{2gy}$



$$\frac{V_1}{\sin\alpha_1} = \frac{V_2}{\sin\alpha_2} \qquad \frac{V}{\sin\alpha} = \text{constant.}$$

$y(A) = 0 \quad y(B) = A$

$$\frac{\sqrt{2gy}}{\sin\alpha} = c \implies y(1 + y'^2) = d$$

example

$(f(x))' = f(x) \quad f(0) = 1$

$$f(x) = 1 + 1\cdot x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots$$

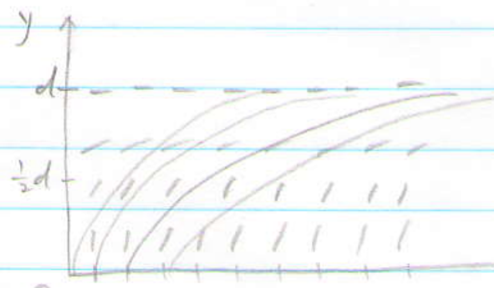
Separation of Variables

$$y(1 + y'^2) = d$$

$$1 + y'^2 = \frac{d}{y}$$

$$y'^2 = \frac{d}{y} - 1 = \frac{d-y}{y}$$

$$y' = \sqrt{\frac{d-y}{y}}$$



$$\boxed{\frac{dy}{dx} = \sqrt{\frac{d-y}{y}}}$$

$$\frac{dy}{\sqrt{\frac{d-y}{y}}} = dx$$

$$\int \sqrt{\frac{y}{d-y}}\, dy = \int dx$$

$$g(y) = x + c$$

$$x = g(y) - c$$

$$y = g^{-1}(x+c)$$

claim

This is a "cycloid"

$$F(t) = \begin{pmatrix} d/2 \cdot t \\ d/2 \end{pmatrix} + \frac{d}{2}\begin{pmatrix} \cos(\frac{3\pi}{2} - t) \\ \sin(\frac{3\pi}{2} - t) \end{pmatrix}$$

$$= \frac{d}{2}\left[\begin{pmatrix} t \\ 1 \end{pmatrix} + \begin{pmatrix} -\sin t \\ -\cos t \end{pmatrix}\right]$$

$$x(t) = \frac{d}{2}(t - \sin t)$$

$$y(t) = \frac{d}{2}(1 - \cos t)$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$= \frac{dy}{dt} \cdot \left(\frac{dx}{dt}\right)^{-1}$$

$$= \frac{d}{2} \sin t \cdot \left(\frac{d}{2}(1 - \cos t)\right)^{-1}$$

$$= \frac{\sin t}{1 - \cos t}$$

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$$\frac{\sin t}{1-\cos t} \stackrel{?}{=} \sqrt{\frac{d-\frac{d}{2}(1-\cos t)}{\frac{d}{2}(1-\cos t)}}$$

$$\frac{\sin^2 t}{(1-\cos t)^2} \stackrel{?}{=} \frac{d-\frac{d}{2}(1-\cos t)}{\frac{d}{2}(1-\cos t)}$$

$$\frac{1-\cos^2 t}{(1-\cos t)^2} \stackrel{?}{=} \frac{1+\cos t}{1-\cos t}$$