0. 0 is a soln
2. The sum of soln is a soln
1. Scalar times soln is a soln
3. dim(soln) = nullity(A)
= n − rank(A)
= # non-pivotal cols

Inhomogeneous case

1. Solns exist iff
$b \in R(A) = \text{col space}(A) \subseteq F^m$
2. If x is a soln of $Ax = b$.
then x′ is also a soln of $Ax' = b$
$\iff x' = x + y$ where $Ay = 0$.

Date. Page.

Pf of 2 $Ax=b$ is given

$\Rightarrow$ Suppose $Ax'=b$ then

$A(x'-x)=b-b=0$.

So if y is $x'-x$, $Ay=0$,

and $x'=x+y$.

$\Leftarrow$ If $Ay=0$ & $x'=x+y$ then

$Ax'=A(x+y)=Ax+Ay=b+0=b$

$Ax=b \xrightleftharpoons[\text{invertible}]{\text{if } E \text{ is}} EAx=Eb$.

$\Updownarrow$

$Cx=d$.

C is obtained from A by row ops & d is obtained from b by same row ops.

$(A|b) \xrightarrow{\text{row ops}} (C|d)$ Do that until C is rref.

Example

$$\left(\begin{array}{cccccc|c} \boxed{x_1} & \boxed{x_2} & x_3 & x_4 & \boxed{x_5} & x_6 & \\ 1 & 0 & 2 & 9 & 0 & e & d_1 \\ 0 & 1 & -3 & 7 & 0 & \pi & d_2 \\ 0 & 0 & 0 & 0 & 1 & 2 & d_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & d_4 \end{array}\right)$$

(C = columns x_3, x_4, x_6; d = d_1, d_2, d_3, d_4)

1. Soln exist. $\Rightarrow$ d_i's in the non-pivotal (zero) rows of C are 0.
2. In this case the x_j's corresponding to the non-pivotal col's can be arbitrarily, and you can still find values unique for the pivotal x_j's that solve the eqn.