UTORid : wuboyan1

P32

1. (f) False.

6 + 6 + 6 = 18

Well done!

eg. $\begin{cases} a+b=1 & ① \\ 3a+3b=1 & ② \end{cases}$

② − ① × 3 :

$\Rightarrow \begin{cases} a+b=1 \\ 0=-2 \end{cases}$

There is no solution for this linear equation.

2. (f) Solution:

$$\begin{cases} x_1+2x_2+6x_3=-1 & ① \\ 2x_1+x_2+x_3=8 & ② \\ 3x_1+x_2-x_3=15 & ③ \\ x_1+3x_2+10x_3=-5 & ④ \end{cases}$$

② − ① × 2 , ③ − ① × 3 , ④ − ① :

$$\Rightarrow \begin{cases} x_1+2x_2+6x_3=-1 & ① \\ -3x_2-11x_3=10 & ② \\ -5x_2-19x_3=18. & ③ \\ x_2+4x_3=-4 & ④ \end{cases}$$

exchange ② and ④ :

$$\Rightarrow \begin{cases} x_1+2x_2+6x_3=-1 & ① \\ x_2+4x_3=-4 & ② \\ -5x_2-19x_3=18 & ③ \\ -3x_2-11x_3=10 & ④ \end{cases}$$

① − ② × 2 , ③ + ② × 5 , ④ + ② × 3 :

$$\Rightarrow \begin{cases} x_1 \quad -2x_3=7 & ① \\ x_2+4x_3=-4 & ② \\ x_3=-2 & ③ \\ x_3=-2 & ④ \end{cases}$$

①+③×2, ②−③×4:

$$\Rightarrow \begin{cases} x_1 = 3 \\ x_2 = 4 \\ x_3 = -2 \end{cases}$$

$$\Rightarrow (x_1, x_2, x_3) = (3, 4, -2)$$

3. (f) Solution: Yes

Now we need to show it and find an expression.

We suppose that

$$(-2,2,2) = a(1,2,-1) + b(-3,-3,3). \quad a, b \in \mathbb{R}$$

$$\Rightarrow (-2,2,2) = (a-3b, 2a-3b, -a+3b)$$

Then we can get a linear equations system:

$$\begin{cases} a\cdot 1 + b\cdot(-3) = -2 \\ a\cdot 2 + b\cdot(-3) = 2 \\ a\cdot(-1) + b\cdot 3 = 2 \end{cases}$$

$$\Rightarrow \begin{cases} a-3b=-2 & ① \\ 2a-3b=2 & ② \\ -a+3b=2 & ③ \end{cases}$$

③+①, ②−①×2:

$$\Rightarrow \begin{cases} a-3b=-2 & ① \\ 3b=6. & ② \\ 0=0 & ③ \end{cases}$$

①+②:

$$\Rightarrow \begin{cases} a=4 \\ b=2 \end{cases}$$

Hence, the first vector can be expressed as a linear combination of the other two.

$$(-2,2,2) = 4(1,2,-1) + 2(-3,-3,3)$$

6/6

4.(f) Solution: No

Now we need to show it.

Suppose that the first polynomial can be expressed as a linear combination of the other two.

That is: $6x^3-3x^2+x+2 = a(x^3-x^2+2x+3)+b(2x^3-3x+1)$

$a, b \in \mathbb{R}$

$\Rightarrow 6x^3-3x^2+x+2 = (a+2b)x^3+(-a)\cdot x^2+(2a-3b)x+(3a+b)$

Then we get a system of linear equations:

$$\begin{cases} a+2b=6 & ① \\ -a = -3 & ② \\ 2a-3b=1 & ③ \\ 3a+b=2 & ④ \end{cases}$$

① + ②, ③ + ② × 2, ④ + ② × 3:

$$\Rightarrow \begin{cases} 2b=3 & ① \\ a=3 & ② \\ -3b=-5 & ③ \\ b=-7 & ④ \end{cases} \Rightarrow \begin{cases} b=\frac{3}{2} & ① \\ a=3 & ② \\ 0=\frac{1}{2} & ③ \\ 0=-\frac{11}{2} & ④ \end{cases}$$

Then, we can find contradictions in ③④.

$\Rightarrow$ There is no solution to this system of linear combination.

Hence, the first polynomial cannot be expressed as a linear combination of the other two.