4. $A \in M_{m \times n}(F)$ $A = (a_{ij}) = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$

Define $L_A : F^n \to F^m$

by $x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \mapsto y = \begin{pmatrix} a_{11}x_1 + a_{12}x_2 + \cdots a_{1n}x_n \\ a_{21}x_1 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + \cdots + a_{mn}x_n \end{pmatrix}$

$y = \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix}$ $y_j = \sum_{i=1}^{n} a_{ji} x_i$

LHS:

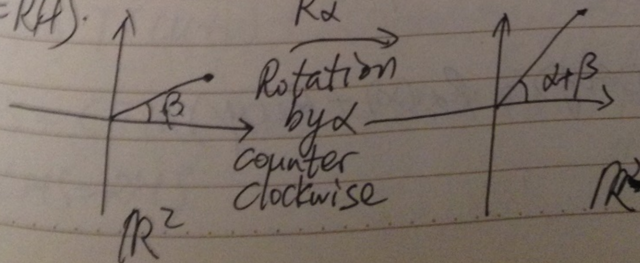
$L(cx+z)_j = \sum_{i=1}^{n} a_{ji}(cx+z)_i$

$= \sum_{i=1}^{n} a_{ji}(cx_i + z_i)$

RHS:

$[cL(x) + L(z)]_j = c\sum_{i=1}^{n} a_{ji} x_i + \sum_{i=1}^{n} a_{ji} z_j$

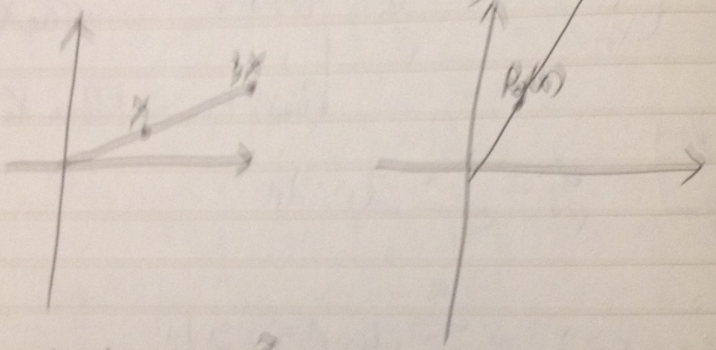
LHS $\stackrel{?}{=}$ RHS.



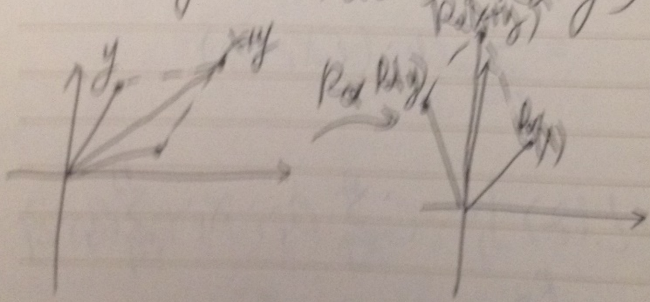
claim R_α is linear

Pf in pictures

2. $R_\alpha(3x) = 3(R_\alpha(x))$



$R_\alpha(x+y) \stackrel{?}{=} R_\alpha(x) + R_\alpha(y)$



$R_\alpha(x+y) = R_\alpha(x) + R_\alpha(y)$