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observation 1

If $D = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$

then $D^p = \begin{pmatrix} \lambda_1^p & 0 \\ 0 & \lambda_2^p \end{pmatrix}$

Pf

$$\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \quad \begin{matrix} \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \\ \begin{pmatrix} \lambda_1^2 & 0 \\ 0 & \lambda_2^2 \end{pmatrix} \end{matrix} \; - \; -$$

observation 2 If $A = CDC^{-1}$ when C is invertible & D is diagonal

then A^p = easy

Pf $A^p = \underbrace{A \cdot A \cdot A \cdots A}_{p} = CDC^{-1}CDC^{-1} \cdots CDC^{-1}$

$= CD^pC^{-1}$

Challenge Find C & D s.t. $A = CDC^{-1}$

$\Leftrightarrow AC = CD$ (*) "diagonalization"

Sol'n $D = \begin{pmatrix}\lambda_1 & 0 \\ 0 & \lambda_2\end{pmatrix}$ $C = \left(v_1 \middle| v_2\right)$

need to find $\lambda_{1,2}$ & $v_{1,2}$ s.t. (*) holds

$$AC = A(v_1|v_2) = (Av_1 | Av_2)$$

$$CD = (v_1|v_2)\begin{pmatrix}\lambda_1 & 0 \\ 0 & \lambda_2\end{pmatrix} = (\lambda_1 v_1 | \lambda_2 v_2)$$

$$\Rightarrow (*) \Leftarrow \forall i \in \{1,2\}\ Av_i = \lambda_i v_i$$

eigenvalue eq'n

eigenvector

eigenvalue

$$\Leftrightarrow Av_i - \lambda_i v_i = 0 \Leftrightarrow (A - \lambda_i I) v_i = 0$$

with $v_i \neq 0$

$N(A - \lambda_i I) \neq \{0\} \Leftrightarrow A - \lambda_i I$ isn't invertible

$\Leftrightarrow |A - \lambda_i I| = 0.$

$$\chi = \begin{vmatrix} 4-\lambda & 3-0 \\ -6-0 & -5-\lambda \end{vmatrix} = (4-\lambda)(-5-\lambda) - 3(-$$

$$= \lambda^2 + \lambda - 2$$

$\Rightarrow \lambda_{1,2}$ are the solutions of the eq'n $\lambda^2+\lambda-2=0$

$\Rightarrow \lambda_1 = 1, \lambda_2 = -2$

Finding V_1: $(A-\lambda_1 I) = 0$

$$\Leftrightarrow \begin{pmatrix} 4-1 & 3-0 \\ -6-0 & -5-1 \end{pmatrix} V_1 = 0.$$

$$\Rightarrow V_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Finding V_2

$$(A-\lambda_2 I)V_2 = 0 \Leftrightarrow \begin{pmatrix} 6 & 3 \\ -6 & -3 \end{pmatrix} V_2 = 0.$$

$$\Rightarrow V_2 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$\Rightarrow D = \begin{pmatrix} 1 & 2 \\ 0 & -2 \end{pmatrix} \quad C = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

$$C^{-1} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$