$$\xrightarrow[C_5 \mathrel{+}= -C_1]{C_4 \mathrel{+}= -C_1} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & -3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{C_4 \mathrel{+}= -C_2} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{C_5 \mathrel{+}= 3C_2} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{C_5 \mathrel{+}= -2C_3} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\text{rank}} 3.$$

<u>claim</u> $\operatorname{rank}(A) = \operatorname{rank}(A^T)$

follows easily from prev. thm

<u>claim</u>

$\operatorname{rank} A = \dim R(A) = \dim($all lin. comb of cols of A

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$= \dim(\text{col space}(A)) = \dim(\text{row space}(A))$

Suppose we're lucky & just by row operations we can reduce A to I_n.

(starting w/ $A \in M_{n \times n}$)

$\Rightarrow (E_4 E_3 E_2 E_1) A = I$

$\Rightarrow A^{-1} = E_4 E_3 E_2 E_1 \cdot I$ ← clever

$\Rightarrow$ If luck is with us, computing A^{-1} is "do to I the same row ops done to A to make it I"

$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ $\qquad$ $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\downarrow$ $\qquad$ $\vdots$

$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $\qquad$ A^{-1}

Even better

$$(A|I) \xrightarrow[\text{ops}]{\text{row}} (I|A^{-1})$$

Example Compute $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^{-1}$

$$\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right) \xrightarrow{r_2 += -3r_1} \begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & -2 & -3 & 0 \end{pmatrix}$$

$$\xrightarrow{r_2 * = -\frac{1}{2}} \begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$$

Claim Can always r/c reduce to the form $\left(\begin{array}{c|c} I_r & 0 \\ \hline 0 & 0 \end{array}\right)$

States If $A=0$ then $A=\left(\begin{array}{c|c} I_0 & 0 \\ \hline 0 & 0 \end{array}\right)$ ✓

otherwise, A has a non-zero entry somewhere

$$A = \begin{pmatrix} & \cdot \neq 0. & \end{pmatrix}$$