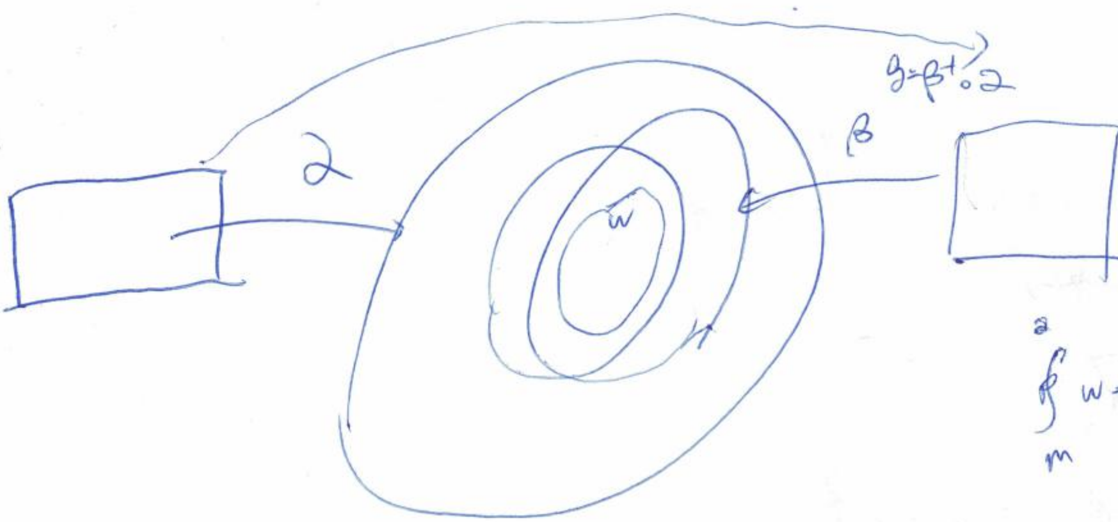


$$\oint_M^{\alpha} w = \pm \oint_M^{\beta} w$$

$$\oint_M^{\alpha} w = \pm \oint_M^{\beta} w$$

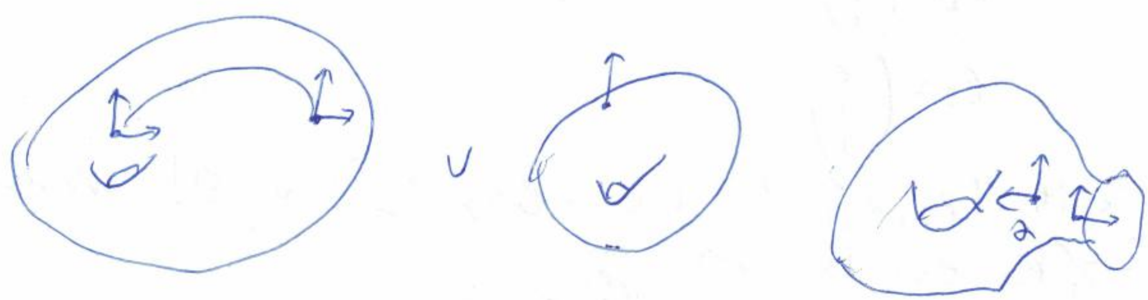
Def. An orientation of M is a continous choice of a positive determinant class of ordered basis of $T_x M$ for $x \in M$.

vocabulary: "orientable"
"oriented"

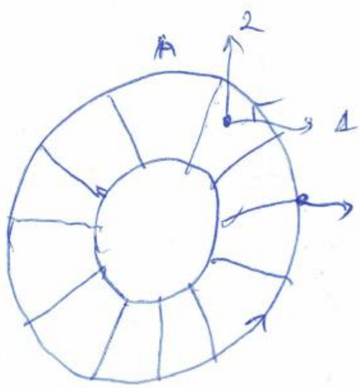
claim: If M is connected, it has 0 or 2 orientations

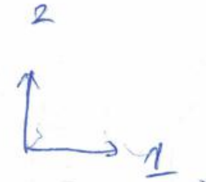
claim: $M^k \subset \mathbb{R}^{k+1}$, orientation iff sides

if M is oriented, then so is ∂M by "∂M"'s orientation is such that prepending to it the outward normal of ∂M gives M's orientation.



Example 1.





$A = \{ x \in \mathbb{R}^2 \mid 1 \leq \|x\| \leq 2 \}$,
oriented by the orientation inherited by
the orientation inherited from the standard orientation
of $\mathbb{R}^2$, (e_1, e_2)

∂A : is oriented with outer boundary counter clock
inner boundary clockwise

Example:

$D^3 = \{ x \in \mathbb{R}^3 : \|x\| \leq 1 \} \subseteq \mathbb{R}^3$, oriented standard way e_1, e_2, e_3

$\partial D^3 = S^2$

How is S^2 oriented at $p = (1, 0, 0)$
How is $T_p S^2$ oriented ?

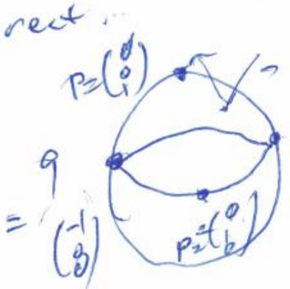
$e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

Is it (e_3, e_2) ? $(e_1, e_3, e_2) \nsim$ std. orientation of

$\Rightarrow (e_2, e_3)$ is correct.

$\underbrace{(-e_1, e_3, e_2)}_{\text{correct.}}$



Exercise: $T_{p_2} S^2$ oriented by (e_3, e_1)
$T_{p_3} S^2$ -"- (e_1, e_2)

$T: V \to W$ is isomorphism of oriented vector spaces. Then T is either
"orientation preserving":
pushing a basis agreeing with orientation of V to a basis agreeing with orientation of W.
"orientation reversing" otherwise.

If $\phi: M^k \to N^k$ is a C^r map of max rank, then it can be orientation preserving positive: $D\phi_p : T_p M \to T_{\phi(p)} N$ is $\forall$ orientation preserving

The composition of two positive map is positive