

MAT 243

Nov 2, 2006.

$$\begin{aligned} \mathcal{L}(V, W) &\rightarrow M_{m \times n}(F) \\ \dim = n &= m \\ \text{basis} = \beta &= \gamma \\ T &\mapsto [T]_{\beta}^{\gamma} = A \end{aligned}$$

$T \mapsto A$
 $S \mapsto B$
 $T + S \mapsto A + B$
 $\gamma T \mapsto \gamma A$

M_{ij} , $i = \text{row}$, $j = \text{col}$

$$A = \left(\begin{array}{c|c} a_{11} & \cdots \\ \hline [V]_D & \cdots \\ a_{mn} \end{array} \right) \Leftrightarrow \forall j \quad T v_j = \sum_{k=1}^m a_{kj} w_k$$

$$\begin{array}{ccc} V & \xrightarrow[S]{B} & W \\ (u_i)_{i=1}^n & \xrightarrow[T]{A} & (w_k)_{k=1}^m \\ T \circ S = ? = C \end{array}$$

$$\Rightarrow A = (a_{kj}) \rightarrow T v_j = \sum_{k=1}^m a_{kj} w_k.$$

$$\Rightarrow B = (b_{ji}) \rightarrow S u_i = \sum_{j=1}^n b_{ji} v_j$$

$$\Rightarrow C = (c_{ki}) \rightarrow (T \circ S)(u_i) = \sum_{k=1}^m c_{ki} w_k$$

$$\begin{aligned} (T \circ S)(u_i) &= T(S(u_i)) = T\left(\sum_{j=1}^n b_{ji} v_j\right) \\ &= \sum_{j=1}^n b_{ji} T(v_j) = \sum_{j=1}^n b_{ji} \sum_{k=1}^m a_{kj} w_k \end{aligned}$$

$$= \sum_{k=1}^m \underbrace{\left(\sum_{j=1}^n a_{kj} b_{ji}\right)}_{C_{ki}} w_k$$

by dist : $\sum_{j=1}^n (d_j w_k) = (\sum d_j) w_k$.

But by Assoc, Commute from
in any order

moral: $C = (C_{ki})$

$$C_{ki} = \sum_{j=1}^n a_{kj} b_{ji}$$

Def'n: In this case we say that C is the "product" of A & B ,
 $C = A \cdot B$ (matrix of $T \circ S$)
 $\downarrow \downarrow$
 $A \quad B$

defined for matrices A and B for which (the number of columns of A) = (the number of rows of B).

Imples

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad 3 \text{ cols}$$

$$B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 0 \\ 1 & 1 & -1 \end{bmatrix} \quad 3 \text{ rows}$$

$$A \in M_{m \times n}, B \in M_{n \times l} \Rightarrow A \cdot B \in M_{m \times l}$$

$A \cdot B$ has as many rows as A and as many columns as B .

↑ 2

↑ 3

$$2 \left\{ \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \textcircled{1} & \textcircled{0} \\ \textcircled{2} & \textcircled{0} & \textcircled{3} \end{bmatrix} \right. \quad C_{23} = \sum_{j=1}^3 a_{2j} \cdot b_{j3} \\ = a_{21} \cdot b_{13} + \\ a_{22} \cdot b_{23} + \\ a_{23} \cdot b_{33} = 4 \cdot 2 + 5 \cdot 0 + 6 \cdot (-1) = 2$$

$$A \cdot B = \begin{pmatrix} A & \boxed{\quad} \end{pmatrix} \quad \begin{pmatrix} B & \boxed{\quad} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 0 \\ 1 & 1 & -1 \end{pmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 4 & 1 & -1 \\ 10 & 1 & 2 \end{bmatrix}$$

$$0, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, +, \cdot$$

Are matrices a field?

NO, but almost. Unless $n=1$

(3)

MAT240

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- 1. x is not always defined.
- 2. many matrices other than $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, have no inverse.
- 3. $A \cdot B \neq B \cdot A$ usually.

eg $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$A \cdot B = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} \quad (\text{rows were swapped})$$

$$B \cdot A = \cancel{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}} \quad \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \quad (\text{columns were swapped})$$

eg. $(2)(3) = (6)$ $M_{1 \times 1} = F$