Nov 2, 2006.

MAT 240

$$\mathcal{L}(V, W) \longrightarrow M_{m\times n}(F) \qquad \begin{matrix} T \to A \\ S \mapsto B \\ T+S \to A+B \\ 7T \Rightarrow 7A \end{matrix}$$

dim = n, m

basis = β, γ

$$T \longmapsto [T]_\beta^\gamma = A$$

$$A = \begin{pmatrix} a_{11} & & a_{1n} \\ [Tv_1]_\gamma & \cdots & [Tv_n]_\gamma \\ a_{m1} & & a_{mn} \end{pmatrix} \iff \forall_j \quad Tv_j = \sum_{k=1}^{m} a_{kj} w_k$$

fundamentals of last class

$$U \xrightarrow[\beta]{S} V \xrightarrow[A]{T} W$$

$(u_i)_{i=1}^{\ell}$, $(v_j)_{j=1}^{n}$, $(w_k)_{k=1}^{m}$

$T\circ S \to \, ? = C$

$$M_{m\times n} \ni A = (a_{kj}) \longrightarrow Tv_j = \sum_{k=1}^{m} a_{kj} w_k$$

$$M_{n\times \ell} \ni B = (b_{ji}) \longrightarrow Su_i = \sum_{j=1}^{n} b_{ji} v_j$$

$$M_{m\times \ell} \ni C = (c_{ki}) \longrightarrow (T\circ S)(u_i) = \sum_{k=1}^{m} \underbrace{c_{ki}}_{?} w_k$$

$$(T\circ S)(u_i) = T(S(u_i)) = T\left(\sum_{j=1}^{n} b_{ji} v_j\right)$$

$$= \sum_{j=1}^{n} b_{ji}\, T(v_j) = \sum_{j=1}^{n} b_{ji} \sum_{k=1}^{m} a_{kj} w_k$$

$$= \sum_{k=1}^{m} \underbrace{\left(\sum_{j=1}^{n} a_{kj} b_{ij}\right)}_{c_{ki}} w_k$$

$$c_{ki} = \sum_{j=1}^{n} a_{kj} b_{ji} \ldots$$

<u>Def</u>: In this case, we say that C is the "product" of A & B, $C = A\cdot B$ (matrix of $T\circ S$) defined for matrices A & B for which (# of columns of A) = (# of rows of B)

<u>Examples</u> $A = \underbrace{\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}}_{3 \text{ cols}}$ $\qquad B = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 0 \\ 1 & 1 & -1 \end{pmatrix}$

$$A \in M_{m\times n},\ B \in M_{n\times \ell} \Rightarrow AB \in M_{m\times \ell}$$

$A\cdot B$ has as many rows as $A \to 2$

as many cells as $B \to 3$

$$2\left\{\underbrace{\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & c_{23} \end{pmatrix}}_{3}\right. \qquad c_{23} = \sum_{j=1}^{3} a_{2j} b_{j3}$$

$$= a_{21}\cdot b_{13} + a_{22}\cdot b_{23} + a_{23}\cdot b_{33} = 4\cdot 2 + 5\cdot 0 + 6(-1) = 2$$

a better way of multiplying matrices

$A \cdot B$

$$\begin{array}{cc} & (B) \\ (A) & \boxed{} \end{array}$$

$$\begin{array}{cc} & \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 0 \\ 1 & 1 & -1 \end{pmatrix} \\ \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} & \begin{pmatrix} 4 & 1 & -1 \\ 10 & 1 & 2 \end{pmatrix} \end{array}$$

ex $(2\cdot 1)+(2\cdot 0)+(3\cdot -1) = -1$

$\underbrace{}_{\text{fundamentals of last class}}$

matrices $\rightarrow$ 0, 1, +, $\cdot$

$\rightarrow$ Are matrices a field?

NO, though almost.

- Why not?

1) $\times$ is not always defined $\rightarrow$ then restrict to 7×7

2) many matrices other than 0 have no inverse

3) $AB \neq BA$

$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\quad B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$$A\cdot B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{array}{c}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \\ \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}\end{array} = \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$$ rows were swapped

$\#$

$$B\cdot A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{array}{c}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}\end{array} = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$$ columns were swapped

$$(2)\begin{array}{c}(3) \\ (6)\end{array} \qquad M_{1\times 1} = F$$