

MAT 240

$$\begin{array}{l} L(v, w) \rightarrow M_{mn}(F) \\ \dim = n^m \\ \text{basis} = \underbrace{\begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \dots & a_{mn} \end{pmatrix}}_T \rightarrow \left[T \right]_B = A \end{array} \quad \left\{ \begin{array}{l} \begin{pmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ T \rightarrow A \\ S \rightarrow B \\ T+S \rightarrow A+B \\ TS \rightarrow TA \end{array} \right. \quad \left. \begin{array}{l} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \\ T \rightarrow A \\ T+S \rightarrow A+B \\ TS \rightarrow TA \end{array} \right. \quad \left. \begin{array}{l} \text{fundamentals of last class} \\ \text{to your related to} \\ \text{fundamentals of linear algebra} \\ \text{containing} \\ \text{fundamentals of linear algebra} \\ \text{fundamentals of linear algebra} \end{array} \right\}$$

$$A = \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ [Tv_1]_Y & \dots & [Tv_n]_Y \\ a_{m1} & \dots & a_{mn} \end{pmatrix} \Leftrightarrow A_j \quad Tv_j = \sum_{k=1}^m a_{kj} w_k \quad \begin{array}{l} \text{containing} \\ \text{fundamentals of linear algebra} \\ \text{fundamentals of linear algebra} \end{array}$$

$$U \xrightarrow[T]{S} V \xrightarrow[T]{S} W \quad \text{fundamentals of linear algebra} \leftarrow \text{fundamentals of linear algebra} \times (S \circ T)$$

$$(U_i)_{i=1}^n \xrightarrow[T \circ S]{S} ? = C \quad \text{fundamentals of linear algebra} \rightarrow \text{fundamentals of linear algebra} \times (S \circ T)$$

$$M_{mn} \rightarrow A = (a_{ij}) \rightarrow Tv_j = \sum_{k=1}^m a_{kj} w_k \quad A \neq BA \quad (\text{E})$$

$$M_{n \times k} \rightarrow B = (b_{ji}) \rightarrow Su_i = \sum_{j=1}^n b_{ji} v_j \quad \begin{pmatrix} S & 1 \\ 0 & I \end{pmatrix} = B \quad \begin{pmatrix} 1 & 0 \\ 0 & I \end{pmatrix} = A$$

$$M_{m \times k} \rightarrow C = (c_{ki}) \rightarrow (T \circ S)(U_i) = \sum_{k=1}^m c_{ki} w_k \quad \begin{pmatrix} S & 1 \\ 0 & I \end{pmatrix} = \begin{pmatrix} S & 1 \\ 0 & I \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & I \end{pmatrix} = B \cdot A$$

$$\begin{aligned} (T \circ S)(U_i) &= T(S(U_i)) = T\left(\sum_{j=1}^n b_{ji} v_j\right) \\ &= \sum_{j=1}^n b_{ji} T(v_j) = \sum_{j=1}^n b_{ji} \sum_{k=1}^m a_{kj} w_k = \begin{pmatrix} S & 1 \\ 0 & I \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & I \end{pmatrix} = A \cdot B \end{aligned}$$

$$C_{ki} = \sum_{j=1}^n a_{kj} b_{ji} \quad \begin{matrix} E \\ \vdots \\ (i) (S) \end{matrix}$$

Def: In this case, we say that C is the "product" of A & B , $C = A \cdot B$ (matrix of $A \cdot B$) defined for matrices A & B for which (# of columns of A) = (# of rows of B)

$$\text{Examples} \quad A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \quad 3 \text{ cols} \quad B = \begin{pmatrix} 1 & 0 & 2 \\ -0 & 1 & 0 \\ 1 & 1 & -1 \end{pmatrix}$$

$$A \in M_{m \times n}, B \in M_{n \times l} \Rightarrow AB \in M_{m \times l}$$

AB has as many rows as $A \rightarrow 2$

as many columns as $B \rightarrow 3$

$$C_{23} = \sum_{j=1}^3 a_{2j} b_{j3} = a_{21} \cdot b_{13} + a_{22} \cdot b_{23} + a_{23} \cdot b_{33} = 4 \cdot 2 + 5 \cdot 0 + 6 \cdot (-1) = 2$$

a better way of multiplying matrices

$$A \cdot B$$

$$(A) \begin{pmatrix} (B) \\ \boxed{\quad} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 0 \\ 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 1 & -1 \\ 10 & 1 & 2 \end{pmatrix}$$

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$$\text{ex } (2 \cdot 1) + (2 \cdot 0) + (3 \cdot -1) = -1 \quad (W, V)$$
$$A \leftarrow [T] \rightarrow T$$

matrices $\rightarrow 0, 1, +, \cdot$

$$\Rightarrow \text{Are matrices a field? } A \leftarrow \begin{pmatrix} \dots & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{pmatrix} \rightarrow \begin{pmatrix} \dots & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{pmatrix} = A$$

NO, though almost.

• Why not?

1) x is not always defined \rightarrow then restrict to 7×7

2) many matrices other than 0 have no inverse

3) $AB \neq BA$

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$A \cdot B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$$

rows were swapped

$$B \cdot A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$$

columns were swapped

$$(2) \begin{pmatrix} 3 \\ 6 \end{pmatrix} M_{1 \times 1} = F$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} = 12$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} = 12$$

To return: $B \cdot A = 12 \cdot 8 \cdot A$ To "subbing" with $x=3$ that you see see see with $= 72$

(B to return to $\#$) divide w/ $8 \cdot A$ return of benefit $(2 \cdot T)$

$$\begin{pmatrix} 8 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} = 8 \quad \begin{pmatrix} 8 & 5 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix} = 8 \quad \text{columns}$$

over ε

$$1 \times M \circ \alpha A \leftarrow M = 8 \cdot \alpha M \circ A$$

$\varepsilon \leftarrow A$ as we know it can't $\neq 0$

$\varepsilon \leftarrow 8 \cdot 2 \cdot 1 / 2 \cdot 8$ return to α

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} = 12$$

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \varepsilon$$

$$S = (1 \cdot 0) + (2 \cdot 2) + (3 \cdot 4) = 24d - 12d + 12d - 12d + 12d - 12d =$$