Mat240 assignment 1 [redacted] 15/16

1. Suppose a & b are nonzero elements of a field F.
Prove that $a^{-1}b^{-1}$ is a multiplicative inverse of ab.

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Proof: To show that $a^{-1}b^{-1}$ is a multiplicative inverse of ab, need to show $(ab)(a^{-1}b^{-1})=1$ ✓

$(ab)(a^{-1}b^{-1}) = (ab)(b^{-1}a^{-1})$ by commutativity of multiplication $xy=yx$

$= a(bb^{-1}a^{-1})$ by associativity of multiplication $(xy)z = x(yz)$ + assoc. once more

$= a(1)a^{-1}$ by definition of multiplicative inverse $xx^{-1} = 1$

$= aa^{-1}$

$= 1$ by $xx^{-1} = 1$ □

2. Write the complex numbers in the form $a + ib$, with $a,b \in \mathbb{R}$:

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2-1. $\frac{1}{2i} + \frac{-2i}{5-i} = \frac{i}{-2} + \frac{-2i(5+i)}{(5-i)(5+i)} = -\frac{1}{2}i + \frac{-10i+2}{26} = \frac{1}{13} + \left(\frac{-23}{26}\right)i$ ✓

2-2. $(1+i)^5 = ((1+i)^2)^2(1+i) = (2i)^2(1+i) = -4(1+i) = (-4) + (-4)i$ ✓

3-1. Prove that the set $F_1 = \{a + b\sqrt{3} : a, b \in \mathbb{Q}\}$ (with the addition & multiplication inherited from $\mathbb{R}$) is a field.

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Proof:

$\mathbb{Q}$ is a field $\Rightarrow$ F_1 is closed under addition & multiplication.

Let $x=a_1+b_1\sqrt{3}$, $y= a_2+b_2\sqrt{3}$, $z= a_3+b_3\sqrt{3}$, be any three elements in F_1.

Commutativity of addition: $x+y=y+x$

$x+y=(a_1+a_2)+(b_1+b_2)\sqrt{3} = y+x$ ✓ $\in F_1$

Commutativity of multiplication: $xy=yx$

$xy = (a_1a_2 + 3b_1b_2) + (a_1b_2 + a_2b_1)\sqrt{3} = yx$ ✓ $\in F_1$

Associativity of addition: $(x+y)+z=x+(y+z)$

$(x+y)+z = (a_1+a_2+a_3) + (b_1+b_2+b_3)\sqrt{3} = x+(y+z)$ ✓

Associativity of multiplication: $(xy)z=x(yz)$

$$(xy)z = a_1a_2a_3 + 3(a_1b_2b_3 + a_2b_1b_3 + a_3b_1b_2) + (b_1a_2a_3 + b_2a_1a_3 + b_3a_1a_2 + 3b_1b_2b_3)\sqrt{3} = x(yz)$$ ✓

Distributivity: $x(y+z)=xy+xz$

$$x(y+z) = a_1a_2 + a_1a_3 + 3b_1b_2 + 3b_1b_3 + (a_1b_2 + a_1b_3 + a_2b_1 + a_3b_1)\sqrt{3} = xy + xz$$ ✓