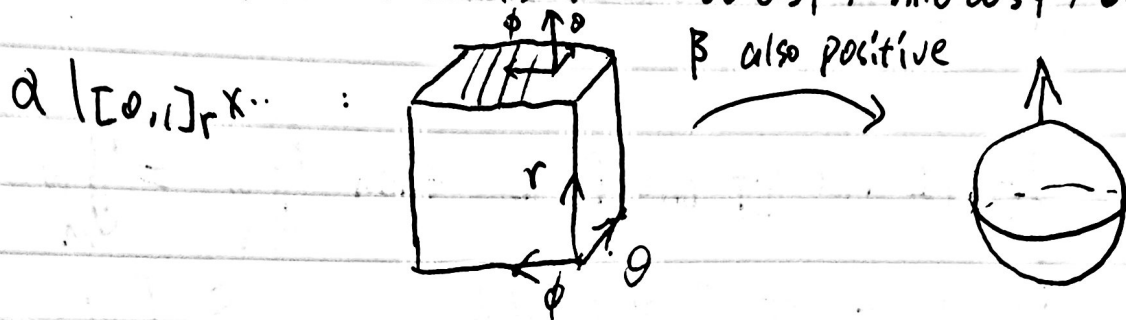


Let $\beta = \alpha|_{r=1}$, $\beta(\theta, \phi) = (\cos\theta \cos\phi, \sin\theta \cos\phi, \sin\phi)$



β is positive, if $[0, 2\pi]_{\theta} \times [-\frac{\pi}{2}, \frac{\pi}{2}]_{\phi}$ is taken with the order of orientation of (θ, ϕ)

$$\int_{S^2} w = \int_{\mathbb{R}^3} w = \int_Q \underbrace{\cos\theta \cos\phi}_{\times} (d \sin\theta \cos\phi) \wedge (d \sin\phi) + 2 \text{ other awful terms}$$

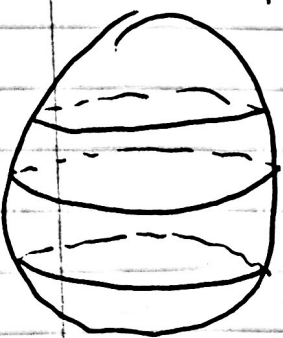
$$Q = [0, 2\pi]_{\theta} \times [-\frac{\pi}{2}, \frac{\pi}{2}]_{\phi} = \int_Q \cos\phi \, d\theta \wedge d\phi$$

$$\downarrow$$

$$= \int_{[0, 2\pi]_{\theta} \times [-\frac{\pi}{2}, \frac{\pi}{2}]_{\phi}} \cos\phi$$

$$= 2\pi \cdot 2 = 4\pi$$

In theory def of $\int_M w$ (M is ~~top~~ compact oriented)



find a pos $\phi_i: M \rightarrow [0, 1]$ smooth

Subordinate to positive charts of M .

* $\text{Supp}(\phi_i) \subset \text{image } \alpha$, α is a positive chart.

* $\sum \phi_i = 1$

* local finiteness

22th Wed March. Hour 066

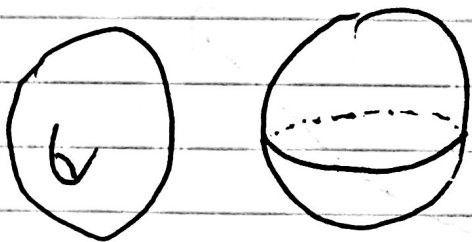
Today: Integration. Stokes's theorem.

Read along: 34.35.37

If M is oriented, α and β on positive and $\text{Supp } W \subset \text{im } \alpha, \text{im } \beta$.
 Then: $\int_{\mathbb{R}^k} \alpha^* W = \int_M W = \int_M \beta^* W = \int_{\mathbb{R}^k} \beta^* W = \int_M W$

$\int_M W$
 \downarrow oriented

in general: Chopped into pieces with measure-0, intersections/exceptions; parametrize and integrate then add together.



Example: Let S^2 be oriented as $\partial D^3 \subset \mathbb{R}^3$, and let $W \in \Omega^2(\mathbb{R}^3)$, given by $W \in x \, dy \wedge dz + (y \, dz \wedge dx + z \, dx \wedge dy)$.

Compute $\int_{S^2} W$ (precisely: $\tau: S^2 \rightarrow \mathbb{R}^3$, $\tau^*: \Omega^2(\mathbb{R}^3) \rightarrow \Omega^2(S^2)$)

$\int_{S^2} \tau^* W \sim \int_{S^2} W$ use $\alpha: [0, \infty) \times [0, 2\pi] \times [-\frac{\pi}{2}, \frac{\pi}{2}] \rightarrow S^2$
 longitude latitude

$\alpha(r, \theta, \phi) = (r \cos \theta \sin \phi, r \sin \theta \cos \phi, r \sin \phi)$

$\det D\alpha = r^2 \cos \phi > 0$, so α is ~~an~~ orientation preserving

So α is orientation preserving positive.

Def: $\int_M w = \sum_{i \in I} \int_M \phi_i w$ makes sense.

Prop: If ϕ_I and ψ_J are POI, then $\phi \int_M w = \int_M w$

Pf: $\phi \int_M w = \sum_{i \in I} \int_M \phi_i w$ def $= \int_M w$

$$= \sum_{i \in I} \int_M \left(\sum_{j \in J} \psi_j \right) \phi_i w \quad \sum_{j \in J} \psi_j = 1$$

$$= \sum_i \sum_j \int_M \psi_j \phi_i w$$

$$= \sum_j \sum_i \int_M \phi_i \psi_j w$$

$$= \sum_j \int_M \left(\sum_i \phi_i \right) \psi_j w$$

$$= \sum_j \int_M \psi_j w = \int_M w$$

Properties: $a_i \in \mathbb{R}$

$$1. \int_M (a_1 w_1 + a_2 w_2) = a_1 \int_M w_1 + a_2 \int_M w_2 \quad \square$$

$$2. \int_{-M} w = - \int_M w = \int_M -w \quad \square$$

$\rightarrow -M$: M taken with opposite orientation

$$\int_{\partial M} w = \int_M dw$$