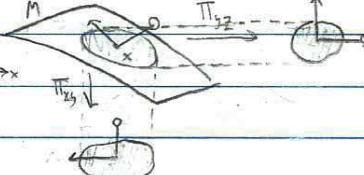
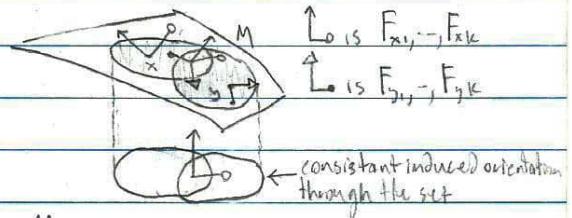


Now, to interpret this result. The 1st thing to notice is that this is independent of coordinate patch, rather, if α, β are coord. patches to x , it only depends on which rows are independent in $D_\alpha(\alpha^i) = D(p\circ\beta^{-1}\alpha)(\beta^i)$
 $= D\beta(\beta^{-1})D(p\circ\alpha)(\alpha^i)$. $D\beta\alpha$ is invertible so if $\{D\alpha^T e_i\}_{i \in I}$ is independent for some $I \in (\frac{n}{k})$,
 $\{D\beta(\beta^{-1})D(p\circ\alpha)(\alpha^i)\}^T e_i = \{D(p\circ\alpha)(\alpha^i)^T [D\beta(\beta^{-1})e_i]\}_{i \in I}$ is independent, so, so is $\{D\beta(\beta^{-1})e_i\}_{i \in I}$.
Thus all coord. patches have the same independent rows at each point.

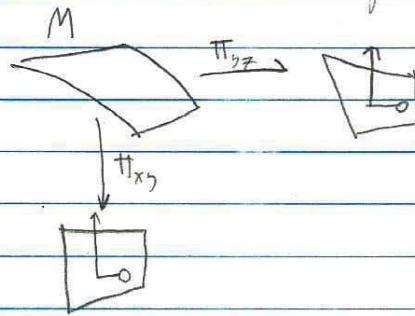
Now, note, $\forall x \in M$, if $y \in M$ $| F_{x_1}, \dots, F_{x_k}$ are defined at x , $(F_{x_i}(x))_{i=1}^k, (F_{y_j}(x))_{j=1}^k \in \mathcal{O}_x$, so $(F_{x_i}(x))_{i=1}^k \sim (F_{y_j}(x))_{j=1}^k$.
As π_{I^*} is injective $\forall I \in (\frac{n}{k})$ for a coordinate patch α , the rows at position I of $D\alpha(x)$ are independent.
 $\pi_{I^*}(F_{x_i}(x)) \sim \pi_{I^*}(F_{y_j}(x))$. So, every $x \in M$ has a small nbhd when projected nicely, the vectorfields
"induce an orientation" of its projection (the induced orientation is the orientation of the projected vectorfield)



We can patch two nbhs together, noting vectorfields defined at the same place induce the same "orientations" just shown:

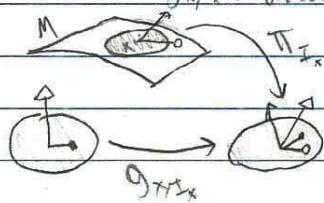


We can patch nbhs together until we "orient" any projection without overlap:
(Clification: what is meant by "orientation" in this case is an orientation at $T_x(\mathbb{R}^k) \forall x \in \pi(M)$)
where if we consider these as orientations of \mathbb{R}^k , they would be the exact same.



The path ahead is clear: we will take a set of patches, make them conform with the projections' orientations & show they overlap positively.

Let $\{\alpha_x\}_{x \in M}$ be a set of patches on M w/ α_x being a patch to $x \in M \quad \forall x \in M$. Restrict the domain & range of each α_x as we've done before so they're connected, the range $\subseteq W_x$ & $g_{x,z}$ is a diffeomorphism for some $I_x \in (\frac{n}{k})$. Let $Dg_{x,z} \neq 0$, continuous & is defined on a connected set,



So, it doesn't change sign, so $g_{x,z}$ carries $(e_i)_{i=1}^k$ to bases of the same orientation over the projection of the domain. If the orientation of \mathbb{R}^k isn't the same as the one induced by the vectorfields w/ α_x , take the -ve of the 1st coordinate so it agrees with the vectorfield's orientation.