eg5 $F_2 \supset F_1$

$\mathbb{C}$ is a V.S. over $\mathbb{R}$.

$\mathbb{R}$ is a V.S over $\mathbb{Q}$

"Galois Theory"

Properties:

1. Cancellation law

addition, multiplication ×2

a. $x+y=y+z$

$\Rightarrow y+z$

m_1. $a\neq 0$ $ax=ay \Rightarrow x=y$

m_2 $x\neq 0$ $ax=bx \Rightarrow a=b$.

Pf: a. Find x' s.t. $x+x'=0$. $(*)$

VS4 $\Rightarrow$ $x'+(x+y)=x'+(x+z)$.

VS2,3 $(*)$ $\Rightarrow$ $y=z$.

M1. $a^{-1}(ax) = a^{-1}(ay)$

VS6 $\Rightarrow (a^{-1}a)x = (a^{-1}a)y$

$1 \cdot x = 1 \cdot y$

VS5 $x = y$.

M2. $ax - bx = (a-b)x = 0$.

Properties 2-7

2. "0_V is unique": If $0' \in V$ satisfies $\forall x$ $0' + x = x$, then $0' = 0_V$

3. "negatives are unique"

If $x+y=0$ & $x+y'=0$, then $y = y'$

(So $(-x)$ makes sense, and $x - y := x + (-y)$ makes sense)

4. $0_F \cdot x = 0_V$

PF: $0x = (0+0)x \overset{VS8}{=} 0x + 0x$

$\Rightarrow 0 = 0x$.

5. $a \cdot 0_V = 0_V$

6. $(-a)\cdot x = -(ax) = a(-x)$

7. If $c \in F$ & $x \in V$, then

$cx = 0 \iff c = 0 \vee (x = 0)$

PF: $\Leftarrow$ Prop 4.5.

$\Rightarrow$ Assume $cx = 0$ if $c = 0$ ✓, $c \neq 0$.

but then $cx = 0 \underset{P5}{=} c \cdot 0 \Rightarrow$

<u>Q</u> suppose $x \in V$, $7 \in F$ is $7x = x \cdot 7$?

Ans <u>no</u>

because RHS is meaningless

In reality always $x \cdot a := a \cdot x$

and then $x \cdot a = a \cdot x$ by definition