

Problem 4. (Dummit and Foote) In $\mathbb{Z}[i]$, find the greatest common divisor of 85 and $1 + 13i$, and express it as a linear combination of these two elements.

From Problem 3 we know that $\mathbb{Z}[i]$ is a Euclidean Domain so can use the euclidean algorithm to find $\gcd(85, 1 + 13i)$:

$$85 = (-6i)(1 + 13i) + (7 + 6i) \quad (1)$$

$$1 + 13i = (i + 1)(7 + 6i) \quad (2)$$

Therefore the greatest common divisor is $7 + 6i$. This can be expressed as a linear combination of 85 and $1 + 13i$ by rearranging the first step to show $85 + 6i(1 + 13i) = 7 + 6i$.