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Example:

$x_{1,2,3} = 0, 1, 3 \quad y_{1,2,3} = 5, 2, 2$

$P(0) = 5 \quad P(1) = 2 = P(3)$

$\tilde{P}_1 = (x - x_2)(x - x_3) = (x^2 - 4x + 3) \quad \tilde{P}_1(x_1) = 3 \quad P_1 = \frac{1}{3}(x^2 - 4x + 3)$

$\tilde{P}_2 = (x - x_1)(x - x_3) = x^2 - 3x \quad P_2 = -\frac{1}{2}(x^2 - 3x)$

$\tilde{P}_3 = (x - x_1)(x - x_2) = x^2 - x \quad P_3 = \frac{1}{6}(x^2 - x)$

$P = 5P_1 + 2P_2 + 2P_3 = x^2 - 4x + 5$

Indeed

$$P(x_k) = \sum_{i=1}^{n+1} y_i P_i(x_k)$$

$$= y_k P_k(x_k) = y_k.$$

<u>Uniqueness</u>

$\beta = \{P_1, P_2, \cdots P_{n+1}\}$ is lin indep.

Indeed, suppose $\sum \alpha_i P_i = 0$.

compute at x_k:

$0 + 0 + \cdots + 0 + \alpha_k P_k(x_k) = 0$

$\Rightarrow \alpha_k = 0 \Rightarrow \forall k \ \alpha_k = 0 \quad \square$

$\dim P_n(F) = n+1 \Rightarrow \beta$ is a basis

$\Rightarrow$ Every $p \in P_n(F)$ can be written (in a unique way) as a l.c of the p_i's

Claim Suppose $q \in P_n(F)$ satisfies $q(x_i) = y_i$ Then $q = p$

Pf By prior claim $\exists \alpha_i \in F$ s.t.

$$q = \sum \alpha_i p_i$$

but $y_k = q(x_k) = \alpha_k$

So $\forall k$ $y_k = \alpha_k$ So

$$q = \sum \alpha_i p_i = \sum y_i p_i = p \quad \square$$

Aside If $\forall p(x_i) = 0$ then p itself is 0.

I.e. If P is a degree n poly not equal to 0, the p has at most n roots.

Linear Transformation

Morphism: A map (function) between two objects of the kind that respects their structure.

0. $L(0_V) = 0_W$
1. $\forall x, y \in V \quad L(x +_V y) = L(x) +_W L(y)$
2. $\forall x \in V \ \forall c \in F \quad L(cx) = c \cdot L(x)$.

Def Let V & W be V.S. over some field F. A linear Transformation from V to W is a map (function) $L: V \to W$ s.t.

1. $\forall x, y \in V \quad L(x+y) = L(x) + L(y)$
2. $\forall x \in V, c \in F \quad L(cx) = cL(x)$.