Solution to Problem 1
Notation: Let (a,b) be the notation used to denote the ideal generated by a and b.

1. Suppose I is principal i.e., (3, x^3-x^2+2x-1) = $(f(x))$, where f(x) = $a_0+a_1x+...+a_nx^n$, $a_i's \in \mathbb{Z}$. Since $3 \in (f(x))$,

$$3 = (a_0 + a_1x + ... + a_nx^n)(g(x)) \quad (1)$$

for some g(x) $\in \mathbb{Z}[x]$, $a_1 = ... = a_n = 0$ (this is because in general, for any two polynomials, deg (f(x)g(x)) = deg (f(x)) + deg (g(x)), so if deg (g(x)) $\neq 0$, then the degree on the right side of equation (1) would be greater than the degree on the left side of equation (1)).

Then (a_0)(g(x)) = 3, so a_0 must divide 3, so $a_0 = 1$ or 3. If $a_0 = 1$, then I = $\mathbb{Z}[x]$. But $I \neq \mathbb{Z}[x]$, since $x \notin I$:

If $x \notin I$, then there would exist $h_1, h_2 \in \mathbb{Z}[x]$ such that

$$x = 3h_1(x) + (x^3 - x^2 + 2x - 1)h_2(x) \quad (2)$$

.

Equation (2) would also be true after we mod 3 on both sides of the equation, so

$$x(\text{mod } 3) = (x^3 - x^2 + 2x - 1)h_2(x)(\text{mod } 3) \quad (3).$$

After we mod three on both sides of equation (3), we can assume that $h_2(x)$ does not have any coefficients that are divisible by 3. If $h_2 = 0$, then we have a contradiction, since $x \neq 0$. Otherwise, deg $(h_2(x)) > 0$, so the degree on the right side of equation (3) is greater than or equal to 3, which is not equal to the degree on the left side of equation (3). This is also a contradiction. Therefore, $x \notin I$.

Since $x \notin I$, $a_0 \neq 1$, so $a_0 = 3$ and f(x) = (3). But there doesn't exists $h(x) \in \mathbb{Z}[x]$ such that $x^3 - x^2 + 3x - 1$ = 3h(x), so $x^3 - x^2 + 3x - 1 \notin (3)$. Contradiction.

Therefore, I is not principal.