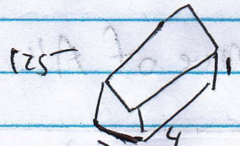
MAT 240 LECTURE 8

Oct 1/14

READ ALONG: 1.3–1.6

RIDDLE ALONG



Can you fit into 10x10x10 cube.

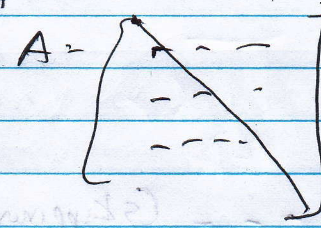
Theorem $W \subset V$ is a subspace iff it is non-empty & "closed under addition & multiplication by a scalar".

Examples a. $W = V$
b. ~~$W \neq \emptyset$~~ $W = \{0\}$ ← can't be empty set b/c has to have 0.

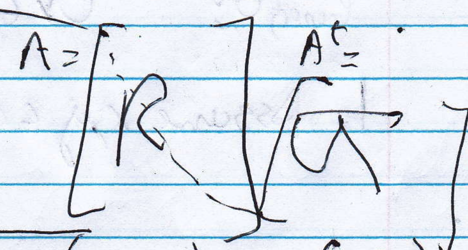
Transpose

$$(A^t)_{ij} = A_{ji}$$

A^t



A^t transpose is a reflection about the above line.



$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$

Example 1

$$W = \{ A \in M_{n\times n}(F) : A^t = A \}$$

$$= \{ A \in M_{n\times n}(F) : \forall ij \; A_{ji} = A_{ij} \}$$

Is W a subspace?

0. Nonempty ✓

1. $A, B \in W$ Is $C = A + B \in W$?

Formally assume $\forall_{ij}$ $A_{ij} = A_{ji}$ & $B_{ij} = B_{ji}$

$\Rightarrow$ $C_{ji} = A_{ji} + B_{ji} = A_{ij} + B_{ij} = C_{ij}$ ✓

$C_{ji} = C_{ij}$ $\therefore C \in W$ ☒

3. $A \in W$ & $c \in F$ $cA \in W$ □

Def'n
$tr(A) = \sum_{i=1}^{n} A_{ii}$

"trace of A"

<u>Example 2</u>

$W = \{A \in M_{n\times n} : \cancel{tr}(A) = 0\}$

0. $W \neq \emptyset$ $\begin{pmatrix} 0 & 0 & 0 & \cdots & 0 \\ - & - & - & & 0 \\ 0 & - & - & - & \end{pmatrix} \in W$. so is $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ (but only 1 required)

1. Assume $A, B \in W$ so $tr(A) = tr(B) = 0$

$C \doteq A + B$

$tr(C) = \sum_{i=1}^{n} C_{ii} = \sum_{i=1}^{n} A_{ii} + B_{ii}$

$= \sum_{i=1}^{n} A_{ii} + \sum_{i=1}^{n} B_{ii}$ (strictly speaking the proof of this being equal involves induction)

$= tr A + tr B = 0 + 0 = 0$ □

2. - - - (skipping proving this part

<u>Discussions</u>: If $W_1, W_2 \subseteq V$ are both subspaces then so is $W_1 \cap W_2 = \{w : w \in W_1 \ \& \ w \in W_2\}$

<u>Proof</u>:
Property 0: $0_V \in W_1$, $0_V \in W_2$. $\Rightarrow$ $0_V \in W_1 \cap W_2$

1. Assume $x, y \in W_1 \cap W_2$, $\Rightarrow$ $x \in W_1$ & ~~x~~ $x \in W_2$, $y \in W_1$ & $y \in W_2$

$\Rightarrow$ $x+y \in W_1$ (b/c W_1 subspace)
~~$\Rightarrow$~~ $x+y \in W_2$ (b/c W_2 subspace)

$\Rightarrow$ $x+y \in W_1 \cap W_2$

2. (skipped)