5. $\dim M_{m \times n}(F) = m \cdot n$.

basis: $\left\{ \begin{pmatrix} 0 0 0 0 0 \cdots 0 \\ 0 0 0 1 0 0 \cdots \\ 0 0 0 0 0 0 \cdots \\ 0 0 0 0 0 \cdots 0 \\ \vdots \\ 0 0 0 0 0 0 0 \end{pmatrix} \right\} = \beta$ $\qquad |\beta| = m \cdot n$.

re $\dim \left\{ \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \right\} = 4$ (all 2×2 matrices)

6. $\dim P_n(F)$

$\{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x^1 + a_0\}$

$\beta = \{x^n, x^{n-1}, \ldots x^0\}$ $\qquad |\beta| = n+1$

7. $\dim P(F)$ = does not make sence. (b/c not finite dimensional)

basis $\beta = (1, x, x^2 \ldots)$ [we found infinite basis]

$|\beta| = \infty$ $\qquad \dim P(F) = \infty$

Lemma ("The replacement Lemma")

Suppose $G, L \subseteq V$ and.

1. G generates : ie $\operatorname{span}(G) = V$
2. L is linearly independent & $|L| = n$ (ie L finite).

Then there exist $R \subseteq G$, $|R| = n$ s.t. $(G \setminus \overset{\text{remove } R}{R}) \cup L$ still spans V. In particular, $|G| \geq n = |L|$