Pf of theorem: Suppose $C \subset \bigcup A_\alpha$ clopen & non-empty.
Then, $\exists \alpha_0$ s.t. $C \cap A_{\alpha_0}$ is non-empty.
and clopen in A_{α_0}
$\Longrightarrow C \supset A_{\alpha_0}$.
$\Longrightarrow C \supset \bigcap A_\alpha$, non empty.
So, $\forall \alpha_1$, $C \cap A_{\alpha_1} \neq \emptyset$.
$\Longrightarrow$ As A_{α_1} is connected, $C \supset A_{\alpha_1}$
But true $\forall \alpha_1$
$\Longrightarrow C \supset \bigcup A_\alpha$