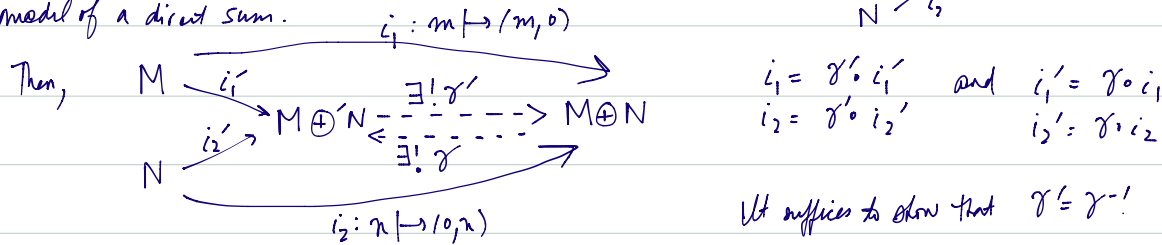


models are isomorphic to the standard model given above. Suppose $M \xrightarrow{i_1} M \oplus N$ is another model of a direct sum.



Now, $i_1 = \gamma' \circ i_1' = \gamma' \circ (\gamma \circ i_1) = (\gamma' \circ \gamma) \circ i_1$ So, $i_1(m) = (\gamma' \circ \gamma)(m, 0)$ since $\gamma \circ \gamma^{-1}$
 $i_2 = \gamma' \circ i_2' = \gamma' \circ (\gamma \circ i_2) = (\gamma' \circ \gamma) \circ i_2$ $i_2(n) = (\gamma' \circ \gamma)(0, n)$

an R -mod morphism, we have $(\gamma' \circ \gamma)(m, 0) + (\gamma' \circ \gamma)(0, n) = (\gamma' \circ \gamma)(m, n)$. But $(m, n) = i_1(m) + i_2(n) = (\gamma' \circ \gamma)(m, n)$. So, $\gamma' \circ \gamma = id_{M \oplus N}$. Similarly, $i_1' = (\gamma \circ \gamma') \circ i_1'$, so $i_2' = (\gamma \circ \gamma') \circ i_2'$

$i_1'(m) = (\gamma \circ \gamma')(i_1'(m))$ so $i_1'(m) + i_2'(n) = (\gamma \circ \gamma')(i_1'(m)) + (\gamma \circ \gamma')(i_2'(n)) = (\gamma \circ \gamma')(i_1'(m) + i_2'(n))$
 $i_2'(n) = (\gamma \circ \gamma')(i_2'(n))$ Thus, to conclude that $\gamma \circ \gamma' = id_{M \oplus N}$, we only NTS that

$M \oplus N = \{ i_1'(m) + i_2'(n) : m \in M, n \in N \}$. " \supseteq " is clear. Fix any element $x \in M \oplus N$.

$\gamma(x) \in M \oplus N$, so $\gamma(x) = (\gamma_1'(x), \gamma_2'(x)) \in M \times N$. NTS: $x = i_1'(m) + i_2'(n)$ for some $m \in M, n \in N$.

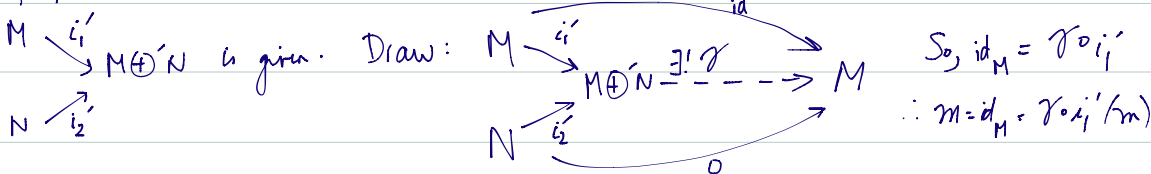
$i_1(\gamma_1'(x)) + i_2(\gamma_2'(x)) = (\gamma_1'(x), \gamma_2'(x)) = \gamma(x) \therefore \gamma \circ \gamma'(x) = x = \gamma(i_1(\gamma_1'(x)) + i_2(\gamma_2'(x)))$

ie. $x = i_1'(m) + i_2'(n)$, as needed. I.e. $M \oplus N \subseteq \{ i_1'(m) + i_2'(n) : m \in M, n \in N \}$.

$\therefore M \oplus N = \{ i_1'(m) + i_2'(n) : m \in M, n \in N \}$ and so we conclude that $\gamma \circ \gamma' = id_{M \oplus N}$.

So, γ is a bijective R -mod morphism, and we conclude that $M \oplus N \cong M \oplus' N$.

Proof of Thm 2. Take $M \oplus N$ to denote our standard model, and suppose another direct sum



Suppose $i_1'(m_1) = i_1'(m_2)$. Then, $m_1 = \gamma \circ i_1'(m_1) = \gamma \circ i_1'(m_2) = m_2$. So i_1' is injective. By symmetry (aka draw a symmetric diagram for i_2') i_2' is injective. \square