Jan. 20th.

Premature Definition. Let $H^k = \{x \in \mathbb{R}^k, x_k \geq 0\}$.

A k-manifold (of class C^r, possibly with boundary) is $M \subset \mathbb{R}^n$ s.t. each $p \in M$ has an open nbhd (in M) s.t. there is an open $U \in \mathbb{R}^k$ and a C^n homeomorphism $\alpha : U \to V$ whose differential is of rank k at every $x \in U$.

The ∂M (boundary M) of M is

$$\left\{ p \in M : \text{For some path } \alpha.\ p = \alpha(q) \text{ where } q \in \partial H^k = \mathbb{R}^{k-1} \times \{0\} \right\}$$

Claim ∂M is also a manifold without boundary

Issure:

1. What does differentiability means on H^k.
2. Could it be the case $\partial M = M$.
3. Why is the claim true.

Def: Let $S \subset \mathbb{R}^k$ & $F : S \to \mathbb{R}$ we say that f is of class C^r on S if there exist an open $U \supset S$ & C^r function $g : U \to \mathbb{R}$, s.t. $g|_S = f$ i.e. $\forall x\ g(x) = f(x)$

Easy fact:

If $f: H^k \to \mathbb{R}^n$. Then $dF(p)$ is well-defined even for $p \in \partial H^k = \mathbb{R}^{k-1} \times \{0\}$

if g_1 and g_2 both extend some set containing H^k. Then $(dg_1)(p) = (dg_2)(p)$ so it makes sense to set $df(p) = dg_1(p)$.

Surprisingly hard fact:

Diffability on a set S is a local property.

If $f: S \to \mathbb{R}$ has the property that for every $p \in S$ there is a nbhd U_p & a differentiable function $g_p: U_p \to \mathbb{R}$ s.t. $F|_{S \cap U_p} = g_p|_{S \cup U_p}$ then f is differentiable on S.

The partition of unity Lemma (Thm):

Given a collection of open sets in $\mathbb{R}^n$ whose overall union is $A = \bigcup_{U \in C} U$, then $\exists \{\phi_i\}$ of non-negative components supported C^∞ fn. s.t.

$$\boxed{\text{support } \{\phi\} = \{x : \phi(x) \neq 0}$$

S.t. 1. suppor $(\phi_i) \subset A$

2. . . .

3. Each supp ϕ_i is cpt

4. $\Sigma \phi_i(x) = 1$