$$\begin{tikzcd} T \arrow[r, phantom] & T' \\ V \arrow[d] \arrow[r, bend left, "R_V"] & V' \arrow[d] \\ W \arrow[r, bend left, "R_W"] & W' \\ T \arrow[r, "R"] & T' \end{tikzcd}$$

s.t. $T' \circ R_V = R_W \circ T$

... also such L, & $L \circ R = I$ $R \circ L = I$ ...

$\dim V = n \iff V \cong F^n$

<u>Goal</u>: $\dim V = n$ $\quad \beta = (v_1 \cdots v_n)$ a basis.

$\dim W = m$ $\quad \gamma = (w_1 \cdots w_m)$.

$\Rightarrow \mathcal{L}(V, W) \cong M_{m \times n}(F)$.

$T \in \mathcal{L}(V, W) \rightsquigarrow [T]_\beta^\gamma = A \in M_{m \times n}(F)$.

The "matrix of T relative β & γ"

knowing $T \iff$ knowing $Tv_i = w_i \in W \iff [Tv_i]_\gamma = \begin{pmatrix} \vdots \end{pmatrix}_m$

$$[T]_\beta^\gamma = A = \underbrace{\left([Tv_1]_\gamma \,\middle|\, [Tv_2]_\gamma \,\middle|\, \cdots \,\middle|\, [Tv_n]_\gamma \right)}_{n} \Big\} m = \underset{\underset{(a_{ij})}{\parallel}}{\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}}$$

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$$\Leftrightarrow T v_i = a_{1i} w_1 + a_{2i} w_2 + \cdots + a_{mi} w_m$$

$$\Leftrightarrow T v_i = \sum_{j=1}^{m} a_{ji} w_j$$

<u>Example</u>

0. $0 \in L(V,W)$ $[0]_\beta^\gamma = \begin{pmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{pmatrix} = 0$

1. $I: \underset{\beta}{V} \to \underset{\beta}{V}$ $\gamma = \beta$ $[I]_\beta^\beta = \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} = I = I_n$

2. $D: P_3(\mathbb{R}) \longrightarrow P_2(\mathbb{R})$

$\beta = (1, x, x^2, x^3)$ $\gamma = (1, x, x^2)$

$$[D]_\beta^\gamma = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$

$D(1) = 0$ $D(x^2) = 2x = 0 \cdot 1 + 2 \cdot x + 0 \cdot x^2$

$D(x) = 1 \cdot 1 + 0 \cdot x + 0 \cdot x^2$ $D(x^3) = 3x^2$.