Date. Page.

$c \in F$ $cx = \begin{pmatrix} cx_1 \\ \vdots \\ cx_n \end{pmatrix}$ $\frac{1}{2}\begin{pmatrix} 8 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ \frac{3}{2} \\ -1 \end{pmatrix}$

claim This satisfies VS1–VS8

eg 1' $F = F_2 = \{0, 1\}$ "bits" $n = 8$

$F^8 = F_2^8 = \left\{ \begin{pmatrix} \quad \end{pmatrix} \right\} = \{(1,0,0,1,1,1,0,1)\}$ = "bytes"

"Bytes make a VS over bits"

$0 \cdot (10011101) = (00000000) = 0_{bytes}$

$$\begin{array}{r} 01011100 \\ +\ 01100111 \\ \hline 00111011 \end{array}$$

checks

VS7 $a(x+y) = a\left(\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}\right)$

$$= a\begin{pmatrix} x_1+y_1 \\ x_2+y_2 \\ \vdots \\ x_n+y_n \end{pmatrix} = \begin{pmatrix} a(x_1+y_1) \\ \vdots \\ a(x_n+y_n) \end{pmatrix} \underset{n\text{ times}}{\overset{F5}{=}} \begin{pmatrix} ax_1+ay_1 \\ \vdots \\ ax_n+ay_n \end{pmatrix}$$

Date. Page.

$$ax+ay = a\begin{pmatrix}x_1\\ \vdots\\ x_n\end{pmatrix} + a\begin{pmatrix}y_1\\ \vdots\\ y_n\end{pmatrix}$$

$$= \begin{pmatrix}ax_1\\ \vdots\\ ax_n\end{pmatrix} + \begin{pmatrix}ay_1\\ \vdots\\ ay_n\end{pmatrix}$$

$$= \begin{pmatrix}ax_1+ay_1\\ \vdots\\ ax_n+ay_n\end{pmatrix}$$

$n=0$ $F^0 = \{(-)\}$ $|F^0| = 1.$

$0_{F^0} = (-)$ $|\{7,-2,0\}| = 3$

"the zero vector space"

$n=1$ $F^1 = \{(x_1) : x_1 \in F\} \longleftrightarrow F$

$(7) \longleftarrow 7$

$(-2) \longleftarrow -2.$

F is a vector space over itself.

Date. Page.

$n=2$, $F=\mathbb{R}$ $V=\mathbb{R}^2=\left\{\begin{pmatrix}x_1\\x_2\end{pmatrix} : x_{1,2}\in\mathbb{R}\right\}$

"The Euclidean plane"

eg 2 $V = M_{m\times n}(F)$

$= \left\{\begin{array}{l}m\times n \text{ matrices}\\ \text{with entries in } F\end{array}\right\}$

$= \left\{A=\begin{pmatrix}a_{11} & \cdots & a_{1n}\\ \vdots & & \vdots\\ a_{m1} & \cdots & a_{mn}\end{pmatrix} : a_{ij}\in F\right\}$

$A=(a_{ij})$

$0_V = 0_{M_{m\times n}(F)} = \begin{pmatrix}0 & 0 & \cdots & 0\\ \vdots & & & \vdots\\ 0 & \cdots & & 0\end{pmatrix}$

$$\begin{pmatrix}a_{11} & \cdots & a_{1n}\\ \vdots & & \vdots\\ a_{m1} & \cdots & a_{mn}\end{pmatrix} + \begin{pmatrix}b_{11} & \cdots & b_{1n}\\ \vdots & & \vdots\\ b_{m1} & \cdots & b_{mn}\end{pmatrix}$$

$$= \begin{pmatrix}a_{11}+b_{11} & \cdots & a_{1n}+b_{1n}\\ \vdots & & \\ a_{m1}+b_{m1} & \cdots & a_{mn}+b_{mn}\end{pmatrix}$$