Oct. 5, 200

MAT240 - Tutorial.

$\boxed{A3}$ - takeup

c) $S \subseteq V$

$\Rightarrow \operatorname{span} S = \overbrace{\text{intersection of all subspaces in } V \text{ containing } S}^{u = \bigcap W,\ V:\ \text{subspace of } V\ S:}$

By Thm. 1.5: span S is a subspace

span S is contained in any subspace of V containi

Pf: ① $\operatorname{span}(S) \subseteq U$

② $U \subseteq \operatorname{span}(S)$

① By thm, $S \subset W$ $\forall$ subspace W containing S

$\Rightarrow \operatorname{span} S \subseteq \bigcap_{W:\text{subspace}} W$

② One of W's is $\operatorname{span}(S)$ by Thm 1.5.

$\Rightarrow U \subseteq \operatorname{span}(S)$

If $S \subseteq Z$ subspace, $\operatorname{span}(S) \subseteq Z$

$\Rightarrow$ all W's contains $\operatorname{span}(S)$

$\Rightarrow U \subseteq \operatorname{span}(S)$

Fact: $\sum_{i=1}^{n} a_i x_i \in \operatorname{span}(S)$ i.e. $x_i \in S$, $a_i \in F$

$x_i \in S \subseteq Z \quad \forall i = 1, \ldots, n$

$\Rightarrow a_i x_i \in Z \quad \because Z$ subspace

$\Rightarrow \sum_{i=1}^{n} a_i x_i \in Z$

2. a) $x_3 + 2x_4 = 4$.

$x_1 - x_2 + 3x_4 = 5$.

$(5 + x_2 - 3x_4,\ x_2,\ 4 - 2x_4,\ x_4)$

$= x_2(1,1,0,0) + x_4(-3,0,-2,1) + (5,0,4,0)$

6. Prove $(110)\ (101)\ (011)$ generates F^3

Show span$\{(110)\ (101)\ (011)\} = \mathbb{F}^3 \to (abc)$

Write (abc) as lin comb of $(110)(101)(011)$.

12. Show subset W is subspace of $V \Leftrightarrow$ span $W = W$

Pf: $(\Rightarrow)$ span $W = W$

i.e. show $W \subseteq$ span W & span $W \subseteq W$

obvious

span $W \subseteq W$

Show $\sum_{i=1}^{n} a_i x_i \in W$ $\because x_i \in W \Rightarrow a_i x_i \in W \Rightarrow \sum_{i=1}^{n} a_i x_i \in W$

$(\Leftarrow)$ By Thm 1.5 span W is always a subspace. Since span $W = W$, W a subspace.

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13. $S_1 \subseteq S_2 \subset V$

$\Rightarrow \text{span}(S_1) \subseteq \text{span}(S_2)$

Pf. $\sum_{i=1}^{n} a_i x_i$, $x_i \in S_1$, $a_i \in F$.

But $x_i \in S_2 \Rightarrow \sum_{i=1}^{n} a_i x_i \in \text{span}(S_2)$

$\text{span}(S_1 \cup S_2) = \text{span}\, S_1 + \text{span}\, S_2$

$\quad = \{x+y \mid x \in \text{span}\, S_1,\ y \in \text{span}\, S_2\}$

Pf: ① $S_i \subseteq S_1 \cup S_2$, $i=1,2$

$\text{span}(S_i \subseteq \text{span}(S_1 \cup S_2)$

$\text{span}\, S_1 + \text{span}\, S_2 \subseteq \text{span}(S_1 \cup S_2)$ ∵ Thm. 1.5

② $\sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n} b_i y_i$ where $x_i \in S_1$, $y_i \in S_2$, $a_i, b_i \in F$.

$\text{span}(S_1 \cap S_2) \subseteq \text{span}(S_1) \cap \text{span}(S_2)$

$S_1 \cap S_2 \subseteq S_i \quad i=1,2$

$\text{span}(S_1 \cap S_2) \subseteq \text{span}\, S_i$

$\therefore \text{span}(S_1 \cap S_2) \subseteq \text{span}\, S_1 \cap \text{span}\, S_2$

Part 2: ~~Ex~~ find examples for which this is true:

$\{0\} \leftarrow \text{span}(S_1 \cap S_2) \subset \text{span}(S_1) \cap \text{span}(S_2) \rightarrow \mathbb{R}^2$

$S_1 = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$ $\quad S_2 = \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\}$

$S_1 \cap S_2 = \emptyset$

16. $S \subseteq V$ whenever $v_1, \dots, v_n \in S$, $\sum_{i=1}^{n} a_i v_i = 0 \Rightarrow a_1 = \dots = a_n = 0$ } S is lin. indep. se

Prove that every $v \in \operatorname{span} S$ ($\neq 0$) can be written uniquely as linear comb of vectors in S.

$v \in \operatorname{span} S$, $v \neq 0$

$\Leftrightarrow v = \sum_{i=1}^{n} a_i x_i \quad x_i \in S$

Suppose $v = \sum_{j=1}^{m} b_j y_j$

$\sum_{j=1}^{m} b_j y_j - \sum_{i=1}^{n} a_i x_i = 0$

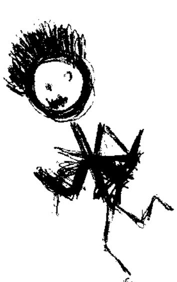
$\Rightarrow b_j = 0 = a_i \ \forall i = 1, \dots, n, \ j = 1, \dots, m$

$\Rightarrow v = 0 \Rightarrow \Leftarrow$.

17. W subspace of V. Under what conditions are there only a finite # of distinct subsets S of W s.t. S generates W?

- $\operatorname{span}\{0\} = \{0\}$ $\quad F = \mathbb{R}, W \neq \{0\}$
- ~~[illegible]~~
- $v \neq 0 \in W$ ~~$\operatorname{span}\{v\} \subseteq W$~~

  $\{av\} \cup W \setminus \operatorname{span}\{av\}$ generates W



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lin indep in $\mathbb{R}^3$:

$\{v_1, v_2, v_3\}$ so that v_1, v_2, v_3 are not in same plane.

$\{v_1, v_2\}$ so that v_1, v_2 are not in same line.

$\{v_1\}$ $v_1 \neq 0$.