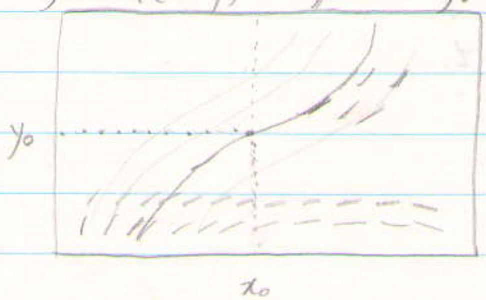
$y' = f(x, y) \quad y(x_0) = y_0$



$y' + P(x)y = g(x)$

Separable Equations

$y' = f(x) \cdot g(y)$

$$\underbrace{\frac{y'}{g(y)}}_{n(y)y'} - \underbrace{f(x)}_{m(x)} = 0 \quad \Rightarrow \quad m(x) + n(y)y' = 0$$

$$y' = \frac{3x^2 + 4x + 2}{2(y-1)}, \quad y(0) = -1$$

$2(y-1)y' - (3x^2 + 4x + 2) = 0$

Easy to remember way : $y' = \frac{dy}{dx}$

$m(x)dx + n(y)dy = 0$

$\int m(x)dx + \int n(y)dy = C$

$M(x) + N(y) = C$

$2(y-1)\,dy = (3x^2 + 4x + 2)\,dx$

$y^2 - 2y = x^3 + 2x^2 + 2x + C$

$(-1)^2 - 2(-1) = 0^3 + 2 \cdot 0^2 + 2 \cdot 0 + C$

$3 = C$

$\therefore C = 3$

$y^2 - 2y - (x^3 + 2x^2 + 2x + 3) = 0 \qquad (\because y(0) = -1)$

$\underline{1 + y^2 - 2y} - (x^3 + 2x^2 + 2x + 4) = 0$

$(y-1)^2 = x^3 + 2x^2 + 2x + 4 \Rightarrow y = 1 - \sqrt{x^3 + 2x^2 + 2x + 4}$

Solve $m(x) + n(y)\,y' = 0$. the justifiable way

Find two new functions $M(x)$ & $N(y)$ s.t.

$$M_x = \frac{\partial M}{\partial x} = M' = m$$

$$N_y = \frac{\partial N}{\partial y} = N' = n$$

The equation becomes $M_x + N_y \cdot y' = 0$

$$\frac{d}{dx}\left(M(x) + N(y(x))\right) = 0$$

$$M(x) + N(y) = c$$

Given $f(x,y)$ $\quad \phi(x_0) = y_0$ & $\phi' = f(x, \phi(x))$

$$m(x) + n(\phi(x))\,\phi' = 0$$

Find M & N as before.

$$M_x + N_y(\phi(x)) \cdot \phi'(x) = 0$$

$$\frac{d}{dx}\left(M(x) + N(\phi(x))\right) = 0$$

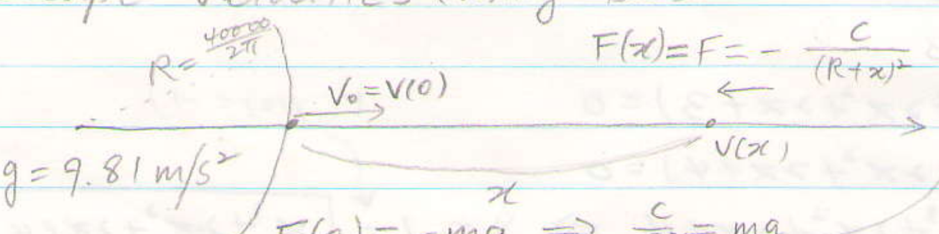
$$M(x) + N(\phi(x)) = c$$

<u>example</u>

① The Brachistochrone : $y' = \underbrace{\sqrt{\frac{d-y}{y}}}_{f(x)\cdot 1}$

② Escape Velocities (using DEs)

$$F(x) = F = -\frac{c}{(R+x)^2} = -\frac{mgR^2}{(R+x)^2}$$



$$F(0) = -mg \Rightarrow \frac{c}{R^2} = mg$$

$F = ma$

$$\Rightarrow -\frac{mgR^2}{(R+x)^2} = m\frac{dv}{dt}$$

$$\frac{dv}{dt} = -\frac{gR^2}{(R+x)^2}, \quad \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} = v'\cdot v$$

$$\Rightarrow v\cdot v' = -\frac{gR^2}{(R+x)^2}$$

$$v\,dv = -\frac{gR^2}{(R+x)^2}dx$$ separated equation!

$$\frac{v^2}{2} = \frac{gR^2}{R+x} + c$$