$$V \ni x \xrightarrow{\text{Isomorphism}} R(x) = [x]_\beta \in F^n$$

the coordinates of x relative to β.

$$x = \sum \alpha_i u_i \Longleftrightarrow [x]_\beta = \begin{pmatrix}\alpha_1 \\ \vdots \\ \alpha_n\end{pmatrix}$$

$$R(x) = \sum \alpha_i e_i$$

Example: In $P_3(\mathbb{R})$

$$[(x-1)^3]_{\{\underset{u_1}{1},\, \underset{u_2}{x},\, \underset{u_3}{x^2},\, \underset{u_4}{x^3}\}} = \begin{pmatrix}-1 \\ 3 \\ -3 \\ 1\end{pmatrix}$$

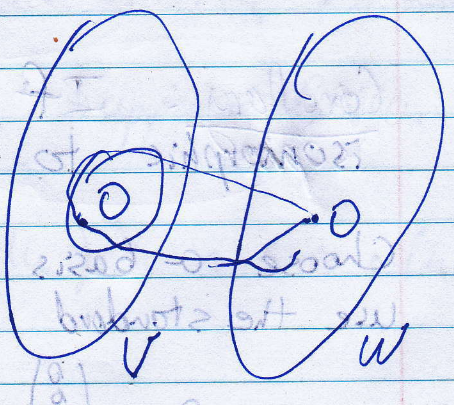
$(x-1)^3 = x^3 - 3x^2 + 3x - 1$

$= u_4 - 3u_3 + 3u_2 - u_1$

Fix a l.t. $T: V \to W$. (V & W v.s. over same basis)

Def^n 1. $N(T) = \operatorname{Ker}(T)$ "Null space" or "Kernel."

Not necessarily isomorphism (if it was it would only map 0 to 0)!



$N(T) = \{ v \in V : T(v) = 0 \}$

2. "Range" "Image" of T

$R(T) = \operatorname{im}(T) = \{ T(v) : v \in V \}$

Preposition/Def^n

1. $N(T) \subseteq V$ is a subspace of V.

$\dim N(T) =$ NULLITY(T)

2. $R(T) \subseteq W$ is a subspace of W.

$\dim R(T) =$ RANK(T)

Micro Proof: $N(T)$: ~~$v_{1,2}$~~ $v_{1,2} \in N(T)$ $Tv_1 = Tv_2 = \vec{0}$

1. $v_1 + v_2 \in N(T)$?

$T(v_1+v_2) = Tv_1 + Tv_2 = 0+0 = 0$ ☑.

2. $R(T)$ $Tv_1, Tv_2 \in R(T)$

$Tv_1 + Tv_2 = T(v_1+v_2) \in R(T)$ ☐

Ex $0 \in \mathcal{L}(V,W)$

$N(0) = V$ nullity$(0) = \dim V$

$R(0) = \{0_W\}$ rank$(0) = 0$

Ex $I_V : V \to V$

$N(I) = \{0\}$ ∴ nullity $= 0$

$R(I) = V$ ~~range~~ rank $= \dim V$