Date 10.24.14 No.

U span $\{u_1, u_2, u_3\}$? Not F^n

In F^4 write 4 eq corresp to $u = c_1u_1 + c_2u_2 + c_3u_3$
4 unknowns
sol'n just first three equ Xenough

e.g $(0,0,1) \in$ span $\{(1,0,0), (0,1,0)\}$ first 2 equ give $c_1 = c_2 = 0$

for $(0,0,1) = c_1(1,0,0) + c_2(0,1,0)$ $\Leftarrow$

3rd eq $1=0$

span $\left(\left\{\begin{bmatrix}1&0\\0&0\end{bmatrix}, \begin{pmatrix}0&1\\1&0\end{pmatrix}, \begin{pmatrix}0&0\\0&1\end{pmatrix}\right\}\right) = \left\{\begin{pmatrix}a&b\\b&c\end{pmatrix} : a, b, c \in F\right\}$

find z st. $z^2 = 2i$ $z = a+ib$, $z^2 = (a+ib)^2 = a^2 - b^2 + i\,2ab$

$\begin{cases} a^2 - b^2 = 0 \\ 2ab = 2 \end{cases}$ $\begin{cases} (a-b)(a+b) = 0 \\ ab = 1 \end{cases}$ $\begin{cases} a = \pm b \\ a = \pm 1 \end{cases}$ $z = 1+i$, $-1-i$ $\begin{cases} a = -b \\ a^2 = -1 \end{cases}$ no sol,

In $F_n = Z_n$, find all sol. of the equ'n

$x^2 = -2$ $x^2 = 9 = 20 = 31 =$ $1^2 = 1$ $2^2 = 4$ $6^2 = (-5)^2 = 25$ $7^2 = (-4)^2 = 16$

$\Updownarrow$

$x^2 - 9 = 0 \Longleftrightarrow (x-3)(x+3) = 0$ $10^2 = (-1)^2 = 1$

$8^2 = (-3)^2 = 9$

If a question is about Z_p, write the final ans as an integer from $0, p-1$.

If f is a p of degree n

if $f(a) = 0$, then $\frac{f(x)}{x-a}$ is also a polynom. (x degree $n-1$)

No.

$S \subset V$ Def Linear dependent means $\alpha u_1 + \dots + \alpha_n u_n = 0$

but $(\alpha_1, \dots, \alpha_n) \neq (0, \dots, 0)$ and u_i are distinct.

then $S_1 \subset S_2 \subset V$, S_1 is lin dep, then S_2 is lin dep.

Then $S_1 \subset S_2 \subset V$ i.e. S_2 is lin ind, then S_2 is S_1

Then if S is lin ind, $V \notin span(S)$, then $S \cup \{V\}$ is lin ind.

Then if finite S is lin ind $(S = \{u_1, \dots u_n\})$

then for any $V \in span(S)$

$V = \alpha_1 u_1 + \dots + \alpha_n u_n$, $\alpha_1, \dots \alpha_n$ are unique

suppose $V = \alpha_1 u_1 + \dots + \alpha_n u_n$

$V = \beta_1 u_1 + \dots \beta_n u_n$

$0 = V - V = \alpha_1 u_1 + \dots + \alpha_n u_n - (\beta_1 u_1 + \dots + \beta_n u_n)$

$0 = (\alpha_1 - \beta_1) u_1 + \dots + (\alpha_n - \beta_n) u_n$

Then by def if lin ind $\alpha_1 - \beta_1 = 0, \Rightarrow \alpha_1 = \beta = 0$

W_1, W_2 are subspaces of a finite dim vect space

s.t. $W_1 \cap W_2 = \{0\}$

$\dim(span(W_1 \cup W_2)) = \dim W_1 + \dim W_2$

let $\beta_1 = \{u_1, \dots, u_n\}$ be a basis for W_1

let $\beta_2 = \{V_1, \dots, V_2\}$ be a basis for W_2

We want to show that $\beta_1 \cup \beta_2$ is a basis for $span(W_1 \cup W_2)$

span (span S) = span S (from class)

Therefore $span(\beta_1 \cup \beta_2) \supset W_1 \cup W_2$

since span (span S) = span S (from class)

Also if $A \subset B$ then span $A \subset$ span B

And span $\beta_1 \cup \beta_2 \supset W_1 \cup W_2$ Therefore

span (β_1 β_2) span ($\beta_1 \cup \beta_2$) $\supset$ span ($W_1 \cup W_2$) $\supset$ span ($\beta_1 \cup \beta_2$)

therefore span ($\beta_1 \cup \beta_2$) = span ($W_1 \cup W_2$)

suppose $a_1 u_1 + \cdots + a_n u_n + b_1 v_1 + \cdots b_{c_1} v_c = 0$

since $W_1 \cap W_2$ {0}

$a_1 u_1 + \cdots + a_n u_n = 0$

$b_1 v_1 + \cdots b_c v_c = 0$

since β_1, β_2 are lind

$a_i = 0, b_i = 0$ $\therefore$ $\beta_1 \cup \beta_2$ is lin ind $\square$